Chapter 3

Linkages in Economic Theory

One prominent aspect of South African industrial development is the continuing strength of sectors close to its mineral endowment. There are numerous potential explanations for this fact, but one I want to shed more light on in this dissertation is the role of linkages between sectors. I interpret linkages narrowly, as the flow of intermediate goods between sectors as recorded in input-output tables (a more elaborate definition of forward and backward linkages follows below). They certainly played a crucial role in South Africa’s early economic development, when mining activities created backward linkages to upstream industries such as chemicals and explosives, and forward linkages to industrial buyers of their output, so called downstream industries. In post-war economic development, the mining boom again galvanised the manufacturing sector. Linkages certainly played a role in industrial policy discussions as well, and the state’s strategic involvement in sectors such as basic chemicals or iron and steel was partly motivated by facilitating linkages between primary extractive industries and downstream manufacturing. The extent of such effects of course has to be evaluated empirically, which I attempt in later chapters.

Before that, a theoretical foundation is necessary. In economic theory, the concept of linkages is an old one which came to prominence thanks to the rise of development economics after World War Two. I will briefly cover this literature (Section 3.1) before turning my attention to a more recent reformulation of the concept: linkages have been formalised in models of the New Economic Geography (NEG)(3.2). I will use such a model to describe
South Africa’s industrial development in Chapter 4 and therefore introduce NEG in a second part of this chapter.

3.1 Linkages in Development Economics

Development Economics is here used as a catch-all to subsume those authors dealing with the long run development of economically backward countries.

German economist Friedrich List may be better known for his arguments on infant industry protection, yet the underlying idea presented by him is equally interesting: in his main work, first published in 1841, he stresses the importance of productive capabilities that are built up in the development process, and how the pursuit of certain industrial activities calls forth production and is productivity enhancing in other sectors of the economy (List, 1950). For him, there is an important difference between a nation using its resources in agricultural production, and one that focuses on manufacturing. The latter is preferable since manufacturing activities lead to productivity increases in all other sectors. Moreover, they trigger processes of institutional, infrastructural and political progress (ibid., 230). Diversification and the emergence of new industries is not a ‘natural’ phenomenon that can be relied on happening in a market economy however. He argues that protective tariffs are necessary to protect ‘infant industries’ from competition—the underlying argument being that productivity increases over time and with scale (ibid., 415ff.). So I find two ideas here that are still very relevant more than a century later: the emphasis on the structural composition of production and linkages between sectors, and the crucial role the state is playing in shaping and influencing this structure.

After 1945, the impending independence of colonies sparked an interest in questions of development that was answered by the nascent discipline of development economics. Often, the concept of linkages was at the very core of these works. In addressing challenges of industrialisation in the European periphery, Rosenstein-Rodan argues for a “large scale planned industrialization” (Rosenstein-Rodan, 1943, 205) in order to enable the exploitation of complementarity of industries. He explicitly refers to external economies as introduced by Marshall (1938) here: the social marginal product of an activity exceeds the private marginal product since it creates linkages—both directly by pushing suppliers of inputs over a certain profitability thresh-
old, and indirectly by raising aggregate demand due to the employment of formerly idle manpower.

Another major contribution came from Myrdal (1957). Rather than perceiving the economy as a system that tends toward a stable equilibrium, he introduced the notion of a vicious (or virtuous) circle where exogenous shocks lead to a self-reinforcing process of cumulative causation. In terms of economic development these processes lead to regional inequalities where certain regions boom thanks to external economies and others decline. Again, policy implications are clear: the state has to provide leadership in identifying projects or industries that can spark positive cumulative processes.

Albert O. Hirschman then went on to explicitly define backward and forward linkages, the former inducing local production of inputs once demand for these inputs reaches a critical scale, the latter providing inputs locally for downstream producers (Hirschman, 1966). He also enriched the concept by stressing two necessary conditions for linkages to work: scale effects—without economies of scale the concept of linkages would be meaningless since every economic activity is linked to many others—and private entrepreneurial or public responsiveness to incentives. Linkages can also be understood as providing investment opportunities and therefore act as guidance for private and state investment (Hirschman, 1981). Here, Hirschman comes very close to ideas of ‘unbalanced growth’ (see for example Streeten, 1959). Hirschman also stresses the importance of transport costs. Linkage effects would not be relevant in their absence, since inputs in production could always be imported without a disadvantage. However, transport costs are significant, particularly for developing countries that often face long distances to large markets. In this formulation, the core elements of a linkage-driven model are already assembled: input-output links between industries, scale economies and significant positive transport costs. A bit later, these effects will be shown to form the core of the NEG model with linkage effects.

Before turning to the model, I want to complete this survey by discussing more recent contributions, even at the risk of over stretching the notion of development economics here. Authors that analyze the rapid rise and industrialisation of East Asia and other developing countries from an institutionalist perspective often stress the role of industrial development and diversification of the manufacturing sector. Moreover, in policy discussions, the East Asian effort is often evoked as a role model for South Africa.
In detailed and empirically rich accounts Wade (1990), Amsden (2001) and others described the interventionist and sector specific industrial policy of East Asian countries that deliberately discriminated between sectors and ‘got the prices wrong’ to steer industrial development. Policy makers used protective trade policies, subsidised credit and export support as carrots to promote investment in certain sectors while at the same time monitoring and evaluating performance with the credible threat of withdrawal of support as stick. Within a neoclassical theoretical framework, economies of scale and scope, learning effects, information and coordination externalities provide justification for targeted intervention and support for specific manufacturing sectors (Chang, 1994; Rodrik, 2004).

The sectors singled out for support were chosen according to dynamic factors. So countries were trying to create (rather than exploit existing) comparative advantage in industries with prospects of “long-term growth in output, profits, and wages” (Wade, 1990, 355). Amsden cites the case of Taiwan, where the government picked industries “based on six criteria: large linkage effects; high market potential; high technology intensity; high value-added; low energy intensity; and low pollution” (Amsden, 2001, 137).

The importance of structural dynamics within developing countries has also resurfaced in recent debates. Ocampo (2005) stresses the importance of a dynamic production structure capable of constantly generating new activities. The importance of sectoral diversification in the long-run development process has also recently been empirically shown in an important paper by Imbs and Wacziarg (2003). Rodrik (2007), in a related effort, then goes on to deduce the necessity of industrial policy to support entrepreneurs, since innovators usually do not reap the full benefits of their risky investments in developing countries.

Very recently, Hausmann and Klinger (2006) and Hausmann and Rodrik (2006) derived a concept they call the product space, where products are positioned to each other according to their closeness. The underlying theoretical assumption is that the production of each good requires specific inputs (skills, intermediate goods). Now some goods are closer to each other than others because the specific inputs required in one’s production can relatively easily be transferred and used in the neighbourly good. The actual pattern of the product space is calculated by evaluating goods exported by more than 100 countries. Two goods are close when their joint exportation by one country occurs frequently. The emerging structure shows that there are products in the core, closely linked to many others, and peripheral
goods that are only weakly connected to other goods. In terms of industrial policy, the implication the authors draw is that governments have to choose which sectors to support by providing those sector-specific inputs that are not made available by markets. Ideally, they choose sectors which are close to the existing production pattern and which move the economy closer to the core of the product space, facilitating further diversification.

To summarise a wide and rather dispersed range of authors, I would argue that both theoretical and empirical work has shown that linkages play a crucial part in the path industrialisation is taking in a country. They provide opportunities for further activities—in the case of a minerals economy such as South Africa these opportunities naturally appear around the resource endowments. This corresponds to the actual development experience of South Africa. Yet linkages alone cannot be relied on to generate investment. Following Hirschman, I understand them more as an opportunity still to be exploited by some agent. Usually this role is taken over by private business, but—and this is what can be learned from the East Asian experience—government should and can play a guiding role in the sectoral allocation of investment when the market cannot be relied upon. It should do so by forging a vision of which path the economy is supposed to take (see for example Chang, 1998)\(^1\); and by picking sectors for support that are on the one hand strongly linked to existing activities, and that on the other hand display dynamic potential as described above.

One last note regarding mineral endowments is necessary at this point. There is a significant body of literature dealing with the particular role natural resources play in the development process. Auty (1993) provides a comprehensive account of the so called resource curse thesis, according to which natural resources are at best a mixed blessing for developing countries. An overvalued exchange rate, high volatility in earnings, and a skewed production structure are all cited as possible and detrimental consequences of a large mining sector. Hirschman argued that the primary sector, including mining activities, is usually weakly linked to the rest of the economy and therefore was skeptical about its role in the industrialisation process as well (Hirschman, 1981, 86). Yet, in a cross-country study of 91 developing countries, Davis (1995) does not find evidence supporting the resource curse thesis—mineral economies did on average outperform the remainder

\(^1\)Rodrik and Hausmann go even further in calling their paper on industrial policy *Doomed to Choose* (Hausmann and Rodrik, 2006)
of countries. Still only one country in his panel managed to lose its status as mineral economy over the observed time period by sufficiently diversifying its export base. “This indicates that [...] it is difficult if not impossible to force diversification of a resource-endowed economy contrary to comparative advantage.” (Davis, 1995, 1772) The South African experience as described above does partially corroborate these findings: the Minerals Energy Complex is rather self-contained in terms of input-output linkages, yet at the same time its very existence gives evidence to the fact that linkage effects were at play in the rise of South Africa’s capital intensive industries.

A linkage-based approach to industrial policy tailored to suit South Africa’s needs would therefore have to take into account the critical role of the mining sector and the Minerals Energy Complex. In a similar vein, Ramos (1998) describes production clusters that develop around initial resource endowments in developing countries. While this is partly a market-based (or natural) process, the theoretical arguments mentioned above also justify government promotion and support for these clusters (see also Porter, 1990).

### 3.2 Linkages in the New Economic Geography

Natural resources and factor endowments of regions can only partly explain the spatial distribution of economic activities. It is easy to see why Johannesburg was founded virtually on top of the rich South African gold fields, yet this alone will hardly be enough to account for the enormous density of economic activity witnessed in Gauteng (the province Johannesburg is located in) today: taking up just 1.4% of the total surface of South Africa, Gauteng accounts for about one third of its GDP (and 8% of the continent’s for that matter!). A large and long standing tradition of geographical economics tries to explain phenomena of ‘second nature’, defined as endogenous forces, brought about by human action, that influence and shape the economic landscape and lead to large regional imbalances and clustering of economic activities (Ottaviano and Thisse, 2004, 2565). Yet, this branch of research played no role in mainstream economic theory. As Krugman (1997, 33) notes, ideas of economic geography do not feature in standard economics textbooks at all. To him, this is due to technical difficulties with modeling economies of scale and the resulting imperfect competition. In brief reviews of this literature, Krugman (1997) and Fujita et al. (2001) argue that ideas of von Thünen, Christaller, Lösch and others did provide insight into is-
sues of spatial economics, but all fell short in terms of the methodological and theoretical rigour expected in mainstream economic analysis (see for example Fujita et al., 2001, 33).

The *New Economic Geography* (henceforth NEG), a now more or less consolidated area of research work that was started and brought to prominence by authors such as Fujita, Venables and Krugman (the seminal reference being Krugman 1991a) provides for a treatment of spatial questions within the framework of neoclassical general equilibrium models. Whether it does justice to the rich tradition of geographical economics is a disputed question that I will not try to answer here (for a critical discussion see for example Martin, 1999). This omission is justifiable because I will concentrate on one particular family of NEG models—the vertical linkages (henceforth VL) models. While being very close to the core NEG work in terms of model structure, they owe a lot of intellectual debt to development economics as discussed in Section 3.1. Many papers in NEG cite the classics of development economics such as Hirschman and Myrdal almost as if they were a canon one has to refer to. Krugman himself dedicates one whole book (Krugman, 1997) to this cause. He explains how the ideas of development economists and economic geographers—interesting and relevant as they were—failed to influence the mainstream of economic theory. The argument is essentially identical to the explanation why geographical economics remained at the periphery of the discipline. Taking Hirschman’s forward and backward linkages as an example, Krugman argues that they only matter if combined with increasing returns to scale in production. Hirschman was aware of this, yet then, economies of scale were difficult to model in a general equilibrium context because they are irreconcilable with perfect competition. Once mathematical modelling became a standard device of economics, the linkages and related arguments involving complementarities or circularity were pushed to the margins of economic theory (Krugman, 1997, 25ff.). Only the innovations introduced in Dixit and Stiglitz (1977) allowed economists to return to these questions within a general equilibrium model framework. NEG models—which are essentially based on the Dixit/Stiglitz framework—allow their reintroduction because they assume economies of scale in production and because they add a spatial dimension (transport costs) to the economy. The two factors combined lead to mathematically told stories that do have a close resemblance to arguments in development economics reviewed above.
I will start this exposition of NEG by briefly presenting the original contribution of Krugman (1991a) verbally. The so-called core-periphery (CP) model is outlined in more detail in Fujita et al. (2001, chap. 4 and 5) and Baldwin et al. (2003, chap. 2). I will then go on to depict the basic VL model in detail—the logic of the model being very close still to the CP model. In the next chapter the multi-industry VL with exogenous growth is presented, complete with parameter values for a minerals economy and simulation results. This model is an extension to the basic model that allows integrating a specific technological structure in the form of input-output tables. An exogenous growth process is then introduced to study the effect of growth on the development and location of industries.

3.2.1 The Logic of NEG: The Core-Periphery Model

The logic of NEG models is probably best explained by starting where it all started: with Krugman’s first core-periphery (CP) model (Krugman, 1991a). It constitutes an attempt to understand how and why the manufacturing sector within a national economy might come to be locally concentrated in a core of the economy, a core which then provides the rest of the economy or the periphery with its manufactured goods. Krugman assumes that there are two sectors, the agricultural or traditional sector and the manufacturing or modern sector. The traditional sector is characterised by constant returns to scale and perfect competition. There are no transport costs and the sector produces a homogeneous good. The manufacturing or modern sector displays decreasing average costs and firms produce a variety of differentiated goods. Transport costs are positive in manufacturing, and decreasing average costs will lead to a limited number of manufacturing production sites. Agriculture on the other hand will be evenly spread. Demand comes from farmers, who are dispersed across the country, and from workers in manufacturing. Since the prevailing technology in manufacturing leads to a concentration in production, final demand from manufacturing wages will be equally concentrated—and a pattern of circular causality emerges. Workers (firms) will move to the core because demand is high, and their very move to the core further increases demand in the core.

The actual spatial distribution of manufacturing activity depends on the parameters of the model: if the manufacturing sector is small and transport costs are high, then manufacturers will locate close to where farmers live, leading to a structure of small towns serving local markets. If then,
over time, manufacturing increases in importance, transport costs fall and economies of scale become stronger, it is easy to see how “the tie of production to the distribution of land will be broken. […] Population will start to concentrate and regions to diverge; once started, this process will feed on itself” (Krugman, 1991a, 487).

Technically, increasing returns to scale are difficult to model, yet a monopolistic competition type market structure as introduced by Dixit and Stiglitz (1977) provides a tractable if very specific formulation. Utility of consumers is of a two-tier structure. They have Cobb-Douglass utility for the two types of goods (agricultural goods and manufactured goods) which of course implies that they spend a constant share of their income ($\mu$ and $1 - \mu$ respectively) on agricultural goods and on the aggregate of manufactured goods. Krugman then goes on to define a sub-utility function for manufactured products. It is of the constant elasticity of substitution (CES) form, so that individuals’ utility increases with the number of varieties offered. The critical parameter here is the elasticity of substitution $\sigma$, where $\sigma > 1$. If $\sigma$ is close to 1, consumers have great love for variety, if $\sigma$ increases, differentiated manufactured products become better substitutes and the desire for variety decreases.

Production in manufacturing on the other hand involves fixed costs, so together with the specific form of demand this market structure implies that each firm will produce only one good—a slightly differentiated product—in one location. The big advantage and simplification of the Dixit/Stiglitz framework is that there is no strategic behaviour of firms. One must assume an infinite (or very large) number of competitors in the market to achieve this result. Then, optimal pricing does not depend on one competitor’s behaviour but is simply a constant mark-up over marginal costs. Since market entry is free, profits will be driven down to zero—the mark-up will just be high enough to cover fixed costs. The constant mark-up has another convenient yet unusual implication: there is one unique level of sales at which operating profit equals fixed costs. Therefore the equilibrium output of a single firm does not depend on market size. A bigger market will lead to an increased number of firms only (Baldwin et al., 2003, 41). One last addition to this technical cook book are iceberg transport costs. It is assumed that a constant fraction of goods ‘melts away’ in transport, in other words transport costs are proportional to marginal production costs. The desirable implications are that no separate transport sector needs to be modelled and firms will engage in mill pricing. They will charge the
same producer price for both consumers in the local and in the remote market, transport costs incurred in the latter are paid for by consumers. With all these features in place, one can analyze the model with numerical simulations.

In the short run, labour is assumed to be immobile. Depending on its initial distribution, real wages will differ in the two regions due to the home market effect (a larger market implies higher sales and higher profits, these are passed on to workers through higher wages) and the market crowding or competition effect (firms in the smaller manufacturing sector face less competition and so pay higher wages). In the long run, workers are mobile, and the migration equation describes workers' movement to regions with higher real wages. When workers move, manufacturing production in the destined region increases—decreasing the cost of living (because more varieties are available locally without incurring transport costs) and thereby further enlarging the gap in real wages. The home market effect is at times referred to as a backward linkage, the cost of living effect as a forward linkage (see for example Ottaviano and Robert-Nicoud, 2006, 114).

The eventual outcome—that is the geographical distribution of manufacturing production, be it a core periphery pattern or a symmetric spread of activity—obviously depends on the initial distribution of the manufacturing sector and on the choice of parameters, particularly the magnitude of trade costs. When trade costs are high, dispersion forces are strong and manufacturing production will take place in both regions. With low trade costs, agglomeration will occur in the region that had some initial (historical) advantage.

3.2.2 Vertical Linkages Models: An Exposition

Since Krugman's initial contribution, a host of different models with similar features and applications have been presented (for an overview and extensive summaries see Fujita et al., 2001; Baldwin et al., 2003). In the context of industrial policy and industrial development, models with vertical linkages (VL) are particularly interesting. The original contributions in this sub-field are from Krugman and Venables (Krugman and Venables, 1995; Venables, 1996). Fujita et al. (2001, chap. 14-16) and Baldwin et al. (2003, chap. 8) present text book versions and, recently, Ottaviano and Robert-Nicoud provided a synthesis of VL models (Ottaviano and Robert-Nicoud, 2006).
What the VL models have in common is that the spatial dynamics result not from movement of labour between regions as in the CP model, but from “intersectoral reallocation of factors within the same location” (Ottaviano and Robert-Nicoud, 2006, 114). These models are set in an international context where labour is assumed to be immobile between regions, yet will move according to wage differentials between the traditional and the manufacturing sector within one location. Firstly, this implies that permanent real wage differentials between countries are possible. Secondly, the driving force of agglomeration now is the manufacturing sector itself which not only produces but also consumes manufacturing goods as inputs. The production function closely resembles the consumption function in the original CP model, which implies that firms spend constant shares of their total production costs on labour and on manufacturing products respectively, with the latter entering in a CES form so that a greater locally available input variety of manufacturing products reduces costs.

It is then possible to redefine the linkage effects. If more firms settle in a region, firms in the region profit from a greater local variety of inputs which are bought without transport costs—a forward linkage. At the same time, more firms in a region imply a higher demand for manufactures as inputs—a backward linkage. While these linkage effects act as agglomerative force, the rising manufacturing wage level in the core relative to the periphery acts as dispersion force which at some point might incite firms to establish themselves in the periphery.

The Formal Model

I will now present in more detail the basic version of the VL model as published in Krugman and Venables (1995) and Fujita et al. (2001, chap. 14). The main difference to these outlines is that I work in a discrete setting from the beginning. For a detailed representation of all derivations the reader is referred to Chapter 4 where a modified and extended version is applied to the research question of this dissertation.

The basic VL model presented here is very close to the original CP model sketched above. There are two regions which are identical—there are no ‘first nature’ or geographical differences between them. Following the literature I will call them North (region $N$) and South (region $S$). In each region, there is a manufacturing sector $M$ which is characterised by increasing returns to scale and positive transport costs (of the iceberg form) between regions, and
a traditional sector \( A \) with constant returns to scale in production and zero transport costs. Goods of both sectors are traded between the two regions.

**Consumer Behaviour**

The representative consumer’s utility is described in a two-tier function. She has a Cobb-Douglass utility function dividing her income between the two types of goods, manufactured and agricultural goods (\( M \) and \( A \) respectively).

\[
U = M^\mu A^{1-\mu}
\]  
(3.1)

A constant share of income \( \mu \) is spent on \( M \), the composite of manufactured goods, and a constant share \( 1 - \mu \) is spent on agricultural goods. As in the CP model, manufacturers produce slightly differentiated products. The composite of these goods is defined as the quantity index \( M \)—a sub-utility function that takes the following form:

\[
M = \left[ \sum_{i=1}^{n} m_i^\rho \right]^{1/\rho}, \quad 0 < \rho < 1
\]  
(3.2)

where \( n \) (the number of varieties) is large. This is a constant elasticity of substitution (CES) utility function. \( \rho \) is indicative of the love for variety of consumers for the different available products \( m_i \). If \( \rho \) is close to zero, consumers value variety greatly, if it is close to one, then the differentiated products become better substitutes and variety is less important. \( n \) represents the number of firms and equals the number of available varieties due to the fact that there are increasing returns in manufacturing—implying that a profit-maximizing firm will produce only one variety. To better understand the CES form, I calculate the elasticity of substitution \( \sigma \) of two varieties.

\[
\sigma = \left[ \frac{\partial(m_2/m_1)}{\partial \text{MRS}} \right] \left[ \frac{\text{MRS}}{(m_2/m_1)} \right]
\]

MRS stands for *marginal rate of substitution* and can be derived as the fraction of marginal utilities of goods \( m_1 \) and \( m_2 \). Utility maximisation implies that the resulting \( \left( \frac{m_2}{m_1} \right)^{1-\rho} \) can be substituted by the price ratio \( \left( \frac{p_1}{p_2} \right) \). Then

\[
\sigma = \frac{\partial \left( \frac{p_1}{p_2} \right)^{1-\rho}}{\partial \left( \frac{p_1}{p_2} \right)} \left( \frac{m_2}{m_1} \right)^{1-\rho}
\]
Simplifying yields

\[ \sigma = \frac{1}{1 - \rho} \]

As indicated above \( \sigma \) is big when \( \rho \) is close to one. In this case, the differentiated manufacturing products are near perfect substitutes and consumers do not value variety highly. When \( \sigma \) is small \( \rho \) is close to zero and consumers display a great love for variety.

Assuming that consumers' income is \( Y \) and \( p_A \) and \( p_i \) are the prices for agricultural and manufactured goods respectively, the budget constraint can be written as follows:

\[ Y = p_A A + \sum_{i=1}^{n} p_i m_i \] (3.3)

The utility maximization problem is solved in two steps. First I derive demand for manufactured goods (equations 3.4 to 3.9), in a second step I will divide income between manufactured goods and agricultural goods (equations 3.10 to 3.11). For manufactured goods, the representative consumer wants to minimize expenditure for achieving a certain utility level \( M \). So the optimisation problem becomes

\[
\min \sum_{i=1}^{n} p_i m_i \quad \text{s.t.} \quad \left[ \sum_{i=1}^{n} m_i^\rho \right]^{1/\rho} = M
\] (3.4)

The marginal rate of substitution equals \( \frac{p_i}{p_j} = (\frac{m_i}{m_j})^{1-\rho} \). From this, an expression for \( m_i \) can be derived.

\[ m_i = m_j \left( \frac{p_j}{p_i} \right)^{\frac{1}{\rho}} \] (3.5)

I substitute 3.5 into the constraint to get compensated demand for \( m_j \)

\[ M = \left[ \sum_{i=1}^{n} \left( m_j \left( \frac{p_j}{p_i} \right)^{\frac{1}{\rho}} \right) \right]^{1/\rho} \]

Bringing \( m_j \) to the left I get

\[ m_j = \frac{p_j^{\frac{1}{\rho-1}}}{\left[ \sum_{i=1}^{n} p_i^{\frac{\rho}{\rho-1}} \right]^{1/\rho}} M \] (3.6)
This is the compensated demand function for \( m_j \). It depends negatively on \( p_j \) and positively on the other varieties’ prices. There is another, insightful way to describe \( m_j \). Multiplying \( m_j \) by its price \( p_j \) and integrating over all varieties, we get total expenditure on the left and a price index on the right.

\[
m_j p_j = p_j^{\frac{\rho}{\rho - 1}} \left[ \sum_{i=1}^{n} p_i^{\frac{\rho}{\rho - 1}} \right]^{-1/\rho} M
\]

Summing up over all varieties \( j=1 \) to \( n \):

\[
\sum_{j=1}^{n} m_j p_j = \left[ \sum_{j=1}^{n} p_j^{\frac{\rho}{\rho - 1}} \right] \left[ \sum_{i=1}^{n} p_i^{\frac{\rho}{\rho - 1}} \right]^{-1/\rho} M
\]

Simplifying yields

\[
\sum_{j=1}^{n} m_j p_j = \left[ \sum_{i=1}^{n} p_i^{\frac{\rho}{\rho - 1}} \right]^{\frac{\rho - 1}{\rho}} M
\]  \hspace{1cm} (3.7)

\( M \) being the quantity composite, the expression on the right can be thought of as a price index. Following Fujita et al. (2001, 47) I name it \( G \).

\[
G \equiv \left[ \sum_{i=1}^{n} p_i^{\frac{\rho}{\rho - 1}} \right]^{\frac{\rho - 1}{\rho}} = \left[ \sum_{i=1}^{n} p_i^{1-\sigma} \right]^{\frac{1}{1-\sigma}}
\]  \hspace{1cm} (3.8)

The price index relates income to utility. It represents the minimum expenditure required to buy one unit of \( M \). It is therefore not an average price but derived from the utility function—and it displays its characteristics as well. To illustrate this, I assume for a moment that all prices are equal. Then, \( G \) becomes \( (np_i^{1-\sigma})^{\frac{1}{1-\sigma}} \) or \( n^{\frac{1}{1-\sigma}} p \). Once the number of varieties \( n \) increases, the price index \( G \) will fall—leading to an increase in consumers’ utility.

Compensated demand can now be rewritten using \( G \).

\[
m_j = \left( \frac{p_j}{G} \right)^{-\sigma} M
\]  \hspace{1cm} (3.9)

The second step of utility maximization consists of dividing total income between expenditure on manufactured and on agricultural goods. Using
equation 3.1 and rewriting the budget constraint with $G$ results in the maximisation problem

$$\max \quad U = M^{\mu} A^{1-\mu} \quad s.t. \quad GM + p^A A = Y \quad (3.10)$$

This standard Cobb-Douglass utility function leads to well known results of agricultural demand $A$ being $(1-\mu)Y/p^A$ and demand for manufactured goods $M = \mu Y/G$. To get consumer demand for each variety of manufactures, this expression for $M$ is substituted into 3.9 to get

$$m_j = \mu Y p_j^{-\sigma} \frac{1}{GG^{-\sigma}} = \mu Y \frac{p_j^{-\sigma}}{G^{-(\sigma-1)}} \quad (3.11)$$

In a last step, indirect utility can be written down by substituting demand for agricultural and manufacturing goods (terms $A$ and $M$) into 3.1:

$$U = \mu^{\mu} (1-\mu)^{1-\mu} Y G^{-\mu} (p^A)^{-(1-\mu)} \quad (3.12)$$

As established further above, an increase in the number of varieties of manufactured goods on offer $n$ reduces $G$, and 3.12 shows that this will increase utility. Varietas delectat.

**Transportation Costs**

As in the CP model, transportation costs in the manufacturing sector are modelled in the iceberg form. That is, a fraction of the good shipped to its destination melts away in the process. So there is no need for a separate transport sector in the model. Crucially, iceberg transport costs also ensure that a firm charges the same price for locally consumed goods and for exports—it engages in mill pricing (Baldwin et al., 2003, 18).

I will come back to the issue of mill pricing in the section on producer behaviour. For now, it suffices to illustrate the concept and its implications for delivery prices. Assume that a manufactured good is produced in $N$. When consumed in the region, there are no transport costs, so the price in the North will be $p_N$. Yet, when it is exported to $S$, only a fraction $1/T_{NS}$ of the good arrives. $T_{NS}$ stands for the amount of the manufactured good that has to be shipped for one unit to arrive in $S$. In terms of prices, a mill price of $p_N$ for one unit implies a delivery price of $p_N T_{NS} = p_{NS}$ in the South.
These costs will obviously affect the price index, which has to be re-written for each region. The price index in $S$ in a discrete setting with $R$ regions would look as follows:

$$G_S = \left[ \sum_{r=1}^{R} n_r (p_r T_r S)^{1-\sigma} \right]^{1/(1-\sigma)} \quad (3.13)$$

It is assumed here that all varieties have the same price (this assumption will be shown to hold later). Then $n_r$ equals the number of firms (i.e. varieties) in each region, and transport costs are taken into consideration for all goods that are shipped in from other regions. So demand in the South for a product from another region, e.g. $N$ can be derived by adapting 3.11:

$$m_{jS} = \mu Y_S (p_N T_{NS})^{-\sigma} G_S^{(\sigma-1)} \quad (3.14)$$

Due to the nature of transport costs, this level of consumption is not equal to the amount that has to be produced in order for $m_{jS}$ to arrive. The parameter $T$ has to be considered. So total sales of a single variety produced in $N$, $q_{jN}$ can be expressed as

$$q_{jN} = \mu \sum_{r=1}^{R} Y_r (p_N T_{Nr})^{-\sigma} G_r^{\sigma-1} T_{Nr} \quad (3.15)$$

3.15 is the sum of demand for the variety $j$ in all regions $r = 1,..,S,N,..R$ and includes the fraction of the good that melted in transport ($T_{Nr}$).

**Producer Behaviour**

As in the CP model, there are two sectors in each region. The agricultural sector is characterised by constant returns to scale in production, hence perfect competition, and no trade costs between regions. The single input in agriculture is labour $l$. To produce one unit of agricultural products $A$, $a_A$ units of labour are needed. The cost function in agriculture therefore equals

$$C_A = w(a_A l) \quad (3.16)$$

where $w$ stands for wages of workers, and it needs $a_A$ units of labour $l$ to produce the agricultural good. Perfect competition leads to marginal cost pricing, so the price of the agricultural good $p^A$ equals $w a_A$. Since trade
in agriculture is free, prices in the sector will equalise over regions, which under certain conditions (as long as there are both sectors in both regions) leads to an equalisation of wages as well.

The manufacturing sector produces a variety of manufacturing products \( q_i \) and uses both labour \( l \) and intermediate goods as inputs. In the CP model, the manufacturing sector used skilled labour only. Note that there is no differentiation between agricultural labour and manufacturing (or skilled) labour in the VL model—in marked contrast to the CP model. Labour therefore can move freely between these two sectors. Additionally, firms in the manufacturing sector face increasing returns to scale. Add to this the preference for variety of consumers and it becomes clear why no firm will produce a variety that is already supplied by one of its competitors. Each firm will produce one distinct variety at one distinct location.

Decreasing average costs are expressed in a fixed cost component \( F \) in the cost function. Both inputs, labour and a CES aggregate of intermediates (identical to the utility formulation of consumers), are used in this fixed input requirement \( F \)—in equal proportion to their use in the variable input requirement \( a_M \). Technology is identical for all varieties and in both regions. One crucial simplification of this version of the VL model is that intermediates stem from the manufacturing sector itself, so basically “manufacturing uses itself (in addition to labour) as an input, that is, that the same aggregate of manufacturing varieties demanded by consumers is also an input into the production of each variety” (Fujita et al., 2001, 241). Linkages effects exist despite there being only one industrial sector! Specifically, inputs are divided in a fixed share \( 1 - \alpha \) of labour and \( \alpha \) of intermediates (the manufacturing composite \( M \)). The latter is the same as the aggregate of manufacturing varieties demanded by consumers.

Assuming for now that fixed costs are covered, then the production function in the manufacturing sector in North equals

\[
q_N = \left[ \alpha^{\alpha}(1 - \alpha)^{\alpha-1} l_N^{1-\alpha} \left[ \sum_{r=1}^{R} n_r q_r^\rho \right]^{\alpha/\rho} \right] / a_M
\]  

(3.17)

\( \alpha^{\alpha}(1 - \alpha)^{\alpha-1} \) are constants necessary to derive a cost function as simple as the one below, \( l_N \) represents labour input, on which \( 1 - \alpha \) of total costs is spent, and \( \left[ \sum_{r} n_r q_r^\rho \right]^{1/\rho} \) represents inputs stemming from intermediates, which takes up a constant share \( \alpha \) of all expenditure. Firms have to spend an equally partitioned amount on the fixed cost component \( F \). \( n \) is the
number of varieties of manufacturing goods produced in each region, $x_{rN}$
the input of each variety produced in region $r$ and $\rho$ again indicates the
extent of preference for variety, this time of firms.

The cost function of the manufacturing sector takes the form

$$C_M = (F + a_M q_N)[w^{1-\alpha}G_N^\alpha]$$

where $G_N$ is the price index in $N$:

$$G_N = \left[ \sum_{r=1}^{R} n_r (p_r T_{rN})^{1-\sigma} \right]^{1/1-\sigma}$$

Due to the specification, the price index for intermediates corresponds to
3.13, the price index for consumers. This is due to the fact that input
demand from firms can be derived in two steps—in line with consumer de-
mand. The demand for a single variety of manufactured goods equals 3.9,
and a constant share of costs of firms $\alpha$ is devoted to manufacturing goods.

So intermediate demand for a single variety (INT) becomes

$$m_{iINT} = \alpha TP_i^{-\sigma} G^{\sigma-1}$$

To arrive at total sales of a variety produced in North, consumer demand
3.15 plus the fraction $T$ lost in transport have to be added. I get

$$q_{iN} = \left[ \mu \sum_{r=1}^{R} Y_r + \alpha \sum_{r=1}^{R} T C_r \right] (p_N T_{NS})^{-\sigma} G_r^{\sigma-1} T_{Nr}$$

Firms maximise profits by equating marginal revenue with marginal costs.
Marginal revenue can be expressed as $p(1 + \frac{1}{c})$. So the elasticity of demand $\epsilon_{q,p}$ has to be calculated.

$$\epsilon_{q,p} = \frac{\partial q}{\partial p}$$

Firms take the price index $G$ as given. This is an important feature of Dixit/
Stiglitz: there is no strategic interaction between competitors. Then it is a
matter of simple derivation to get $-\sigma$ as the elasticity of demand. To arrive
at optimal price setting, marginal revenue and marginal costs can now be
equated.

$$MR = MC \quad \Rightarrow \quad p(1 + \frac{1}{-\sigma}) = (w^{1-\alpha}G^\alpha)a_M$$
I replace $\sigma$ with $\frac{1}{1-\rho}$ to get the profit maximizing price

$$p = \frac{(w^{1-\alpha}G^{\alpha})a_M}{\rho}$$  \hfill (3.23)

Firms set prices by charging a constant mark up over marginal costs.

Analysing 3.21 also helps to understand the concept of mill pricing. The elasticity of demand will be $-\sigma$ for domestic demand as well as for demand in other locations since transport costs enter proportional to the value of goods. For this reason, firms will charge the same mill price $p$ for local and foreign consumption. The end price in the remote region corresponds to the respective marginal costs and therefore has to include transport costs and can be written as $pT$.

The technology of firms stated in the production function 3.17 already constitutes the first source of linkage effects. Since intermediates enter in a CES form, firms profit from a greater variety of intermediates available to them locally—it will lead to a decrease in their price index. One can think of this linkage as a forward linkage. A greater number of suppliers of intermediate goods in a region reduces costs for firms using these intermediates in their production.

In Dixit/Stiglitz, there is free entry into markets, so profits are driven down to zero. I use this condition to define the equilibrium size of a manufacturing firm.

$$\Pi = pq - (w^{1-\alpha}G^{\alpha})[F + a_M q]$$  \hfill (3.24)

where

$$p = \frac{(w^{1-\alpha}G^{\alpha})a_M}{\rho}$$

Replacing $\rho$ with $\frac{\sigma - 1}{\sigma}$ and setting $\Pi = 0$ yields equilibrium firm size $q^*$

$$q^* = \frac{F(\sigma - 1)}{a_M}$$  \hfill (3.25)

Note that 3.25 implies that equilibrium firm size does not depend on the size of the market, but only on cost parameters and consumers’ love for variety. So an increase in market size leads to an increase in the number of firms in the market only (and thereby further diversifies manufacturing supply) but leaves firm size unaffected.
It is now possible to re-write expenditures on manufactures $E$ in a particular location. It consists of demand from consumers and of intermediate demand by firms.

$$E_r = \mu Y + \alpha n_r p_r q^*$$  \hspace{1cm} (3.26)

The zero profit condition ensures that the total value of a firm’s production ($pq^*$) equals its total costs. So with $\alpha$ the part of costs devoted to inputs, the second term on the right is simply all firms’ expenditure on intermediates. This expenditure represents the second source of linkages, so called backward linkages. The more firms operate in one location, the higher their intermediate demand, and this demand raises total demand for manufacturing products.

To find the number of firms in the market, one has to define the size of the market—in other words, labour supply. For simplicity, let labour supply equal 1 in both countries. As described above, labour is intersectorally mobile, so farm workers can move into manufacturing and vice versa, but labour is internationally immobile. Take $\lambda$ to be the share of labour that is engaged in manufacturing. Above, I defined the value of total manufacturing in a country, and since a constant share $(1 - \alpha)$ of costs is spent on workers, the total wage bill in manufacturing can be derived.

$$w\lambda = (1 - \alpha)n pq^*$$  \hspace{1cm} (3.27)

From 3.25 we know equilibrium firm size, so $n^*$, the equilibrium number of firms, becomes

$$n^* = \frac{\lambda w \rho}{(w^{1-\alpha}G^{\alpha})F(\sigma - 1) (1 - \alpha)}$$  \hspace{1cm} (3.28)

Using $n^*$ and the profit maximizing price 3.23, the price index for region North 3.19 in a 2-region setting can be rewritten as

$$G_N^{1-\sigma} = \left[ \lambda_N w_N^{1-\sigma(1-\alpha)}G_N^{\sigma-\sigma\alpha} \frac{p^\sigma a_M^{1-\sigma}}{F(\sigma - 1)(1 - \alpha)} \right] + $$

$$\left[ \lambda_S w_S^{1-\sigma(1-\alpha)}G_S^{\sigma-\sigma\alpha} \frac{p^\sigma a_M^{1-\sigma}}{F(\sigma - 1)(1 - \alpha)} \right] T^{1-\sigma}$$  \hspace{1cm} (3.29)

This equation is not very handy, so following Fujita et al. (2001, 242f.) I introduce some normalizations. I choose units so that the marginal input requirement $a_M$ equals $\rho$ and equilibrium output of a firm $q^*$ equals $1/(1 - \alpha)$. 

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Then the profit maximizing price and the price index can be reformulated to get

\[ p = w^{1-\alpha}G^\alpha \]  

(3.30)

for a much simplified price and

\[ G_N^{1-\sigma} = \lambda_N w_N^{1-\sigma(1-\alpha)}G_N^{-\sigma\alpha} + \lambda_S w_S^{1-\sigma(1-\alpha)}G_S^{-\sigma\alpha}T^{1-\sigma} \]  

(3.31)

as the simplified price index for the region \( N \) in a two-regions setting.

**Short Run Equilibrium**

I can now define the short run equilibrium. It is important to understand that entry is free even in the short run, so profits will still be zero. Yet in the short run, labour is immobile, so wages can differ between the two regions.

Using the normalisations introduced above, the price indices for regions \( N \) and \( S \) become

\[ G_N^{1-\sigma} = \lambda_N w_N^{1-\sigma(1-\alpha)}G_N^{-\sigma\alpha} + \lambda_S w_S^{1-\sigma(1-\alpha)}G_S^{-\sigma\alpha}T^{1-\sigma} \]  

(3.32)

and

\[ G_S^{1-\sigma} = \lambda_S w_S^{1-\sigma(1-\alpha)}G_S^{-\sigma\alpha} + \lambda_N w_N^{1-\sigma(1-\alpha)}G_N^{-\sigma\alpha}T^{1-\sigma} \]  

(3.33)

respectively. 3.32 and 3.33 are crucial expressions and embody central aspects and features of the model. If for example \( \lambda \), the share of manufacturing in a region, increases in \( N \), then the price index \( G_N \) will fall—and firms profit from cheaper intermediate inputs. This is what I defined above as a *forward linkage*. Also, these equations show how the price indices depend on both wages and, via intermediates, on the price indices themselves.

Now that both consumer and producer behaviour are known, it is time to look at market clearing conditions. Market clearance is ensured in an indirect way by defining the so-called *wage equations*. They determine the level of wages manufacturing firms are allowed to pay for them just to break even—in other words, the wage level at which the zero profit condition is fulfilled. To arrive at the wage equations, I first equate \( q^* \), equilibrium output by firms, with demand for a particular manufacturing variety as described in 3.21.

\[ q^* = \left[ \mu \sum_{r=1}^{R} Y_r + \alpha \sum_{r=1}^{R} TC_r \right] (p_NT_{NS})^{-\sigma} G_r^{\sigma-1} T_{Nr} \]  

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Using the normalisations introduced above, $E$ for total expenditure on manufactured goods, and bringing wages (which are directly linked to prices via mark-up pricing) to the left I get the wage equations for regions $N$ and $S$:

\[
\frac{(w_N^{1-\alpha}G_N^{\alpha})^\sigma}{1-\alpha} = E_N G_N^{\sigma-1} + E_S G_S^{\sigma-1} T^{1-\sigma} \tag{3.34}
\]

\[
\frac{(w_S^{1-\alpha}G_S^{\alpha})^\sigma}{1-\alpha} = E_S G_S^{\sigma-1} + E_N G_N^{\sigma-1} T^{1-\sigma} \tag{3.35}
\]

For firms to break even, they have to charge a certain price. This price depends on both the wage level and the price index, the latter coming into the picture due to the existence of intermediate inputs. Together with the normalisation for $q^*$ this accounts for the left side of the wage equation. On the right, it is obvious that spending on manufactures comes from consumers and firms (both included in $E$), from the home market and, additionally, from exports to the other region.

The wage equation implies that increased spending in a region will push up wages since demand for a single variety will increase—a backward linkage. The expansion of a sector also leads to a greater variety of industrial goods on offer, which decreases the costs of industrial inputs. This effect shows up in $G$ on the left side of the wage equation and leads to higher wages as well—it can be interpreted as a forward linkage. A countering effect is the increased competition that follows a larger variety on offer. This effect is represented in $G$ on the right hand side of the equation and tends to reduce the manufacturing wage. Changes in the manufacturing wage in relation to changes in the regional distribution of manufacturing activity as described here will be the driving force of the adjustment process from the short-run to the long-run equilibrium.

Total expenditure $E$ is defined as

\[
E_N = \mu Y_N + \frac{\alpha w_N \lambda_N}{1-\alpha}, \quad E_S = \mu Y_S + \frac{\alpha w_S \lambda_S}{1-\alpha} \tag{3.36}
\]

where expenditure comes from consumers (first term) and from the industrial sector itself via intermediate demand. Consumers spend a share $\mu$ of their income on manufactured goods, while firms’ respective expenditure is of the share $\alpha$, derived from the cost function. From 3.27 we know that the value of manufacturing output is $npq^*$ and that it can be expressed as $\frac{w^*}{1-\alpha}$, leading to the second term of the expression.
Income $Y$ comes from both sectors. The total labour force equals 1, so income generated in manufacturing is simply $w\lambda$. Turning to income generated in agriculture, we know that labour is the sole factor of production. For reasons of simplicity we assume an extremely simple and linear production function: $q_A = (1 - \lambda)$, where $(1 - \lambda)$ is the amount of labour used in agriculture. $a_A$ from equation 3.16 is set to unity. Since the agricultural good acts as the numéraire, the marginal product of labour in the sector and hence the agricultural wage are also unity.

$$Y_N = w_N \lambda N + (1 - \lambda N), \quad Y_S = w_S \lambda S + (1 - \lambda S) \quad (3.37)$$

The wage gap $v$ between sectors is

$$v_N = w_N - 1, \quad v_S = w_S - 1 \quad (3.38)$$

Together, equations 3.32 to 3.38 characterise the short run equilibrium.

**Long Run Equilibria**

In the short run, wages will differ between sectors, so there is an incentive for workers to move to the sector with higher wages. The assumption is simply that workers move out of agriculture and work in the manufacturing sector when wages in manufacturing are higher or $v > 0$. The law of motion is defined as follows:

$$\dot{\lambda} = v(\lambda(1 - \lambda)) \quad (3.39)$$

So there are three possible long run equilibria in a region, in which there is no incentive for workers to leave their sector: wages are equalised across sectors—both sectors will operate; all production is concentrated in manufacturing and manufacturing wages are higher or equal to (a virtual) agricultural wage; or lastly all production is concentrated in agriculture and wages are higher or at least equal to (a virtual) manufacturing wage.

$$w = 1, \quad \lambda \in (0, 1),$$

$$w \geq 1, \quad \lambda = 1,$$

$$w \leq 1, \quad \lambda = 0 \quad (3.40)$$

These equilibria depend on the initial distribution of manufacturing activities and on parameter values. The most common representation of the model is to look at equilibria depending on the significance of transport.
costs. High transport costs usually lead to a symmetric long-run equilibrium, meaning that manufacturing production takes place in both regions. This also means that both countries are active in the traditional and in the modern sector, meaning that wages all over are unity. The reason for this spatial distribution is that in the case of high transport costs, it pays to be close to consumers, and farmers—equally spread over the regions—are a significant part of final demand.

When transport costs fall, the importance of being close to these ‘remote’ consumers erodes and the symmetric equilibrium becomes unstable. So if manufacturing production increases only slightly in one region, this will increase the manufacturing wage in the region, leading to a further movement of workers into manufacturing in the region. Put more technically, forward and backward linkages (outweighing the competition effect) trigger an agglomeration process and lead to industrial concentration in one region, while the other specializes in agriculture—the core-periphery outcome.

### 3.2.3 A Simulation Exercise

To illustrate the characteristics of the model, I will present some results from a numerical simulation done with Mathematica. First, equations 3.32 to 3.38 are numerically solved in a two-regions setting to arrive at the short run equilibrium. Then, workers’ movement (triggered by wage differentials) is simulated as a discrete adjustment process.

The short run equilibrium depends critically on parameter values which are set as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.4</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Setting $T = 3$ means that the simulation starts in a situation where transport costs are high. If $\lambda_N$ and $\lambda_S$ are set to 0.7 and 0.1 respectively (representing an initial spatial distribution of manufacturing of the core-

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2Were the initial values 0 and 1, there would be no movement of workers. However, this turns out not to be a stable equilibrium. To test for stability, I use starting values slightly different from the extreme values.
periphery type) then the wage gap $v_N$ is negative and $v_S$ positive. There is an incentive in the North for workers to leave the manufacturing sector and move into agriculture and vice versa in the South.

The law of motion is formulated as follows in the simulation:\footnote{The slight difference to Equation 3.39 is the inclusion of parameter $\gamma$ which—set at 0.5—slows the movement of workers. It is necessary in a discrete simulation exercise such as this one to prevent overshooting.}

$$\dot{\lambda} = \gamma v(\lambda(1 - \lambda))$$

Movement of workers leads to a symmetric long run equilibrium—$\lambda_N$ and $\lambda_S$ are both 0.4, so manufacturing production is equally split between the two regions. This result is not sensitive to the initial values of $\lambda_r$ but driven by high transport costs. This becomes obvious when one lets transport costs fall. Using the symmetric outcome as starting values for $\lambda_r$, the symmetric equilibrium remains the long run outcome up until $T = 1.95$. Once $T$ falls below this threshold (the so-called break point), the symmetric equilibrium becomes unstable. The region that gains a slight advantage in manufacturing will experience specialisation in the modern sector, the other region will deindustrialise. The outcome is the core-periphery equilibrium. As laid out above, the strength of linkages causes the concentration of manufacturing activity in one region.

With very low transport costs ($T = 1.5$) any initial distribution will lead to a core-periphery outcome in the long run. Which region ends up as the industrialised core, and which one as the periphery depends on the initial conditions. A slight advantage for the North in the beginning suffices to lead to total agglomeration of manufacturing in the core in the long run. Interestingly, the core-periphery outcome remains the long run equilibrium up until a level of transport costs of $T = 2.2$. Only at $T = 2.25$ will an initial core-periphery pattern become unsustainable and the two regions move to a symmetric outcome. This latter level of transport costs is called the sustain point in the literature (Baldwin et al., 2003, 30f.). Break and sustain points can also be derived algebraically. A detailed derivation can be found in Fujita et al. (2001, 248f.).

To describe the model, we can therefore identify three different outcomes, depending on the level of transport costs. Given the parameter values assumed in this simulation, the core-periphery outcome will always be the long run equilibrium for low transport costs ($1 < T < 1.9$). At an intermediate...
Manufacturing Shares, $T = 1.5$

Figure 3.1: Manufacturing employment at $T = 1.5$

level ($1.95 < T < 2.2$), the long run equilibrium depends on the initial conditions. A core-periphery structure will give rise to a core-periphery long run equilibrium, a symmetric distribution of activities will lead to a symmetric long run equilibrium. With high transport costs ($T > 2.25$), the equilibrium will always be symmetric, no matter the initial conditions.

There are various ways to graphically present the results of such a simulation exercise. The best known is the Tomahawk diagram which displays stable equilibria in relation to transport costs. I choose a different way here, depicting the share of each region’s labour force active in manufacturing were the manufacturing wage to be unity in one of the regions, hence equal to the agricultural wage. In a situation of low transport costs—from above we know that this implies a core-periphery outcome—the stable long run equilibria are the corner solutions, when either $w_N = 1$ or $w_S = 1$ intersect with the axes.

To interpret Figure 3.1, consider $w_N = 1$ first. This curve depicts all combinations of $\lambda_N$ and $\lambda_S$ at which manufacturing wages in the North are unity, or at which there is no incentive for workers in the North to leave the sector they are active in at the moment. This is the case when all manufacturing is concentrated in the North, but also in the symmetric...
equilibrium ($\lambda_N = \lambda_S = 0.4$) and all other combinations on the curve. At any point to the right of $w_N = 1$, the manufacturing wage is less than unity and $\lambda_N$ would decrease because of wage incentives, at any point to the left $\lambda_N$ is higher than unity and workers would move into the modern sector. The same is true for the South (where any point above $w_S = 1$ implies manufacturing wages below unity and leads to a fall in $\lambda_S$), and it becomes clear why only the corner solutions are stable equilibria: any point to the left (to the right) of both lines would lead to an increase (to a decrease) of manufacturing activity in both regions. If the two regions are on a hypothetical point $\lambda_N = 0.7, \lambda_S = 0.1$ (on $w_N = 1$), then wages of manufacturing workers in the South are below unity and they would move into agriculture. $\lambda_S$ would decrease, $\lambda_N$ would increase further until the two regions arrive in the core-periphery solution. The same would hold for a comparable situation close to the other corner solution. Finally, the symmetric equilibrium is not stable since a slight movement of $\lambda$ in any region would trigger a specialisation process and lead to one of the two corner solutions.

In the case of high transport costs ($T = 3$) we already know to expect the symmetric equilibrium. The graphic representation of wage developments confirms this (see Figure 3.2). Only the symmetric equilibrium is stable, an inspection of any other point in the graph will confirm the tendency of workers to move in a way that leads to an equal distribution of manufacturing activity.

The latter case is certainly the most interesting. At intermediate transport costs ($T = 2.15$ in this simulation) we saw above that the outcome depends on the initial distribution of manufacturing activity—it might be symmetric or a core-periphery pattern. Figure 3.3 shows the results in terms of wages. There are five equilibria overall now, the two corner solutions, the symmetric equilibrium and two intermediate solutions. Only the first three turn out to be stable. To see why, take the curve $w_S = 1$. If the regions are situated on the left-most part of the curve ($\lambda_S$ between 0 and 0.2), then Northern workers have an incentive to leave manufacturing since manufacturing wages are below unity. In this case (and at very low levels of $\lambda_S$) the corner solution—the core-periphery pattern—will prevail.

In contrast, if we presume that the two regions are on the second from left part of $w_S = 1$, then workers in the North earn wages in manufacturing that exceed agricultural wages. This will lead them to move into the modern sector, increasing $\lambda_N$ and decreasing $\lambda_S$. The economies move towards
the symmetric equilibrium. This exercise can easily be continued, but the outcome should be clear by now: depending on the starting position, the two regions will either end up in a core-periphery pattern (with the core in the North if it has a strong initial advantage or vice versa) or in a symmetric equilibrium (if the initial position is sufficiently close to the symmetric position).

3.2.4 Conclusion

This simple vertical linkages model reestablishes the results of the Core-Periphery model: depending on initial conditions, the geographical distribution of economic activity changes endogenously, potentially leading to processes of industrialisation and de-industrialisation respectively. The ultimate cause for such specialisation processes\(^4\) are the backward and forward linkages that manufacturing firms create locally once they set up shop in a region. This is the crucial difference to the Core-Periphery model, where ag-

\(^4\)The term specialisation seems more appropriate than the term agglomeration in this case, since both regions concentrate on one particular activity in the core-periphery outcome, rather than one region taking up all economic activity.
glomeration processes are driven by the geographical movement of workers and their purchasing power.

Since linkages play an important role in South African industrial development, the vertical linkages model provides an entry point into a discussion of its industrial development. However the limitation of the model to one homogeneous industrial sector would not permit to take the specific structure of South African industry into consideration. Therefore, the next chapter introduces a vertical linkages model with multiple sectors and also provides simulation analysis that takes into account some of the peculiar features of the South African economy that were described in Chapter 2.