Chapter 4

A Simulation of Linkage Effects

South African industrial development has led to an economic structure that is rather peculiar given its factor endowments: production is capital-intensive and concentrated to a large extent in capital-intensive sectors. This pattern is even stronger when one looks at international competitiveness and the country’s exports. This is surprising and requires explanation when considering that South Africa has an abundance of unskilled labour at its disposal.

In Chapter 2 I argued that its mineral endowment, and the industrial development that was triggered by it, are responsible for this structure. Linkages between the mining sector and closely related industrial sectors shaped the economy while labour-intensive manufacturing sectors remained weakly developed throughout history. Almost from the beginning of the mining operations, input sectors such as explosives and chemicals, electricity generation and even mining machinery were started in the country. Equally important, the foreign exchange earned through the export of minerals allowed policy makers to support the emergence of an inward-oriented manufacturing sector that produced consumer goods for the growing domestic market. While this pattern was established in the period between the First and the Second World War, it continued (and became more accentuated) after 1945 and can serve to describe even today’s economy to a large extent.

The tight links between mining and a range of other sectors—electricity, which is overwhelmingly based on coal mining, chemical industries, and basic
metals such as steel—has led Fine and Rustomjee (1996) to the coining of the term Minerals Energy Complex. It is a notion that is picked up in this work and is useful when thinking of the South African industrial development process as driven by linkages between these sectors.

A second notion that is often evoked is South Africa’s intermediate position in international trade relations. It struggles to compete with developed countries in the export of high-tech and skills-intensive goods, and it faces equally stiff competition in labour-intensive goods, where low-wage regions particularly from Asia undercut its competitiveness. The former is caused by a shortage of skills, the latter by relatively high wage costs. South Africa is a middle income country with a corresponding cost structure, its history of oppression and exploitation of African labour make wage cuts a politically extremely sensitive issue, and its minerals exports also put an upward bias on the exchange rate of the Rand.

I take these two core features of the economy—linkages between mining and certain industrial activities shaping industrial development, and South Africa’s intermediate position in the global economy, where it faces competition from both ends of the skills spectrum—and try to reproduce them in a vertical linkages model with multiple sectors (that allow to model linkages between them) and three regions (that can cater not only for a core and a periphery, but also an intermediate region).

4.1 The Multi-Industry Model

The logic of vertical linkages models was explained above. For the model presented here, I will mostly draw on work done by Puga and Venables (1996, 1999). They provide a particularly interesting framework that lends itself to applications on development economies and industrial development. They set up a multi-country and multi-industry framework where each industry is imperfectly competitive and industries are linked by an input-output structure. Starting with an initial core periphery distribution of manufacturing, they then add an exogenous growth process that expresses itself in increased demand for manufactured goods. At first, strong linkage effects let new firms in the core satisfy additional demand, driving up wages and increasing inequality between regions. At a certain wage level, it becomes profitable for firms to forego benefits from linkages in the centre, and take advantage of low wages in the periphery by moving there. They thereby create linkages and
trigger a self-sustaining industrialisation process in the periphery—a story reminiscent of old ideas like circular causality and cumulative causation.

In the 1996 paper, Puga and Venables simulate this process for a set of hypothetical scenarios where industries differ in terms of labour intensity, upstream and downstream position or linkage strength and then look into which industries leave an existing agglomeration first. In the 1999 application, they aggregate their input-output structure from the actual South Korean input-output matrix. Forslid et al. (2002) provide an application in a European context, albeit with a focus on changing trade costs rather than growth, and Fujita et al. (2001, Chapter 15) have an exposition of the Puga/Venables model in their overview as well.

4.1.1 Logic of the Model

One striking feature of world economic development in the last few decades is the rapid growth of Asian economies such as South Korea and Taiwan. Much of their success is attributed to the formidable performance of their manufacturing firms which managed to penetrate world markets and export increasingly sophisticated goods to the rest of the world. Often, their growth is analysed in terms of national policies, and their experience has influenced discussions on industrial and more broadly economic policy in developing countries. In chapter 6, which tackles very recent industrial policy changes in South Africa, this influence will become very visible. However, the vertical linkages model as presented here aims to provide a different explanation. Rather than looking at country differences, it is assumed that countries are similar in their technology and their factor endowments. The differences in industrial activity are explained by agglomeration effects that result from some initial advantage and forward and backward linkages.

There are multiple regions, and each of the regions possesses a traditional and a multitude of manufacturing sectors. These sectors are linked to each other via an input-output matrix. A by now well known combination of factors allows for agglomeration processes: the existence of positive transport costs, increasing returns to scale in production and linkages between sectors. Because of positive transport costs, it is important for firms to be located close to consumers (which in this setting can be both final consumers or households, and other firms that use their product as an input) and to the producers of input that they themselves use in production. In combination with increasing returns to scale, this ensures that firms will choose
a singular location close to the main market, rather than spreading production geographically as they could with constant returns to scale. Lastly, input-output linkages represent the sectoral connections that reinforce agglomeration. Once an upstream industry is located in a certain region, it becomes very attractive for downstream sectors to set up shop close by.

One region has—for some historical region that is not necessarily explained in the model—an initial advantage and therefore becomes the core of manufacturing activity. All manufacturing production is concentrated in the region because of the backward and forward linkages at play. The other regions represent the periphery and are specialised in agriculture. The existing agglomeration and the linkages are a strong incentive for any new manufacturing firms to settle in the core. However, there is an opposing force: wages in the core will be higher than in the periphery. As in the model laid out above, wages rise with manufacturing activity in a region.

These two opposing tendencies provide for interesting developments once a growth process is introduced. It is modeled very simply as an increase in demand for manufacturing goods that results from exogenous technical progress and a related increase in labour productivity. With higher demand for manufacturing goods, additional firms will come into operation. At first, they will locate in the core because of linkages. However, more activity widens the wage gap, and at one point it will pay for a firm to start its business in the periphery, where it forgoes proximity to suppliers and customers, but employs labour at lower wages. Unsurprisingly, firms in labour-intensive sectors and firms with weak linkages to other firms will move first. With growth continuing, the region that attracted these first movers will develop a manufacturing sector over time, creating its own linkages, but also experiencing rising wages—it will turn into a second core. This process repeats itself with the other regions, describing a development process where one region after the other experiences development spurts which lead to industrialisation and higher wage levels.

In order to represent South Africa's situation as closely as is possible in this model framework, I will introduce a number of variations below. There will be three regions, one representing the core with a concentration of manufacturing activity, and two peripheral ones. Out of five different manufacturing sectors, one will use an additional input in production—it can be thought of as minerals. This sector therefore represents the mining and mineral beneficiation sectors. The minerals good will be exogenously priced, and is only available in one of the two peripheral states. Apart from this
differing minerals endowment, the regions will be identical as was the case above. Starting from this position, I will try to show how a growth process that causes additional manufacturing production will lead to a particular pattern in the emergence of manufacturing activity in the periphery: firms in sectors that are closely linked to mining through input-output linkages will set up shop in the region which disposes of minerals—a trivial result. However, firms in labour intensive sectors that have little connections to the mining industry will evade the mineral-rich region because of higher wages, and they will rather settle in the third region, which thus starts a process of labour-intensive industrialisation.

As a result, the resource-rich country remains caught in a development path that is not labour-intensive. Linkage effects and wage differentials prevent the attraction of more labour-absorbing industries. This seems to be a reasonable, albeit by necessity extremely simplified, representation of South African industrial development.

4.1.2 The Formal Model

The model resembles the one presented in Chapter 3, but there are a number of important differences. I will therefore present it in detail and step by step, following the overall structure of the earlier presentation, but with a focus on those aspects that have changed.

Consumer Behaviour

The representative consumer draws utility from the consumption of a homogeneous agricultural good, and from industrial goods of five different sectors $s$. A minimum amount of the agricultural good has to be consumed—this amount $\bar{Y}$ can be thought of as a subsistence level of food consumption. All income $Y$ up to $\bar{Y}$ is spent on the agricultural good by the representative consumer. For income above $\bar{Y}$, she allocates constant shares of her income to each of the six sectors. The utility function is thus a linear expenditure system as developed by Stone (1954).

Moreover, within each of the five manufacturing sectors $s = 1...5$, consumers display a preference for variety: they draw utility from consuming different varieties of the manufactured good produced in the sector. $m_i$ is the quantity of a variety consumed, and there is a range of $n$ varieties on offer in a sector, which is determined in the model.
\[ M_s \] Composite index of the quantity of consumption of manufactured goods produced in sector \( s \)

\[ m_{is} \] Consumption of a variety \( i \) of the manufactured good produced in sector \( s \)

\( A \) Consumption of the agricultural good

\( p_{A}, p_{is} \) Price of the agricultural and the manufactured good respectively

\( Y \) Income of the representative consumer

\( \bar{Y} \) Subsistence level of consumption of the agricultural good

\( \mu_s \) Share of income above \( \bar{Y} \) spent on sector \( s \)

\( 0 < \rho < 1 \) Indicator for the love for variety; the smaller it is, the greater is the preference for variety

\( \sigma \) Elasticity of substitution between varieties, the bigger it is, the greater is the preference for variety

\( G_s \) Price index in sector \( s \)

\( T_{NS} \) Transport costs for shipping a good from region \( N \) to region \( S \)

\( \delta_s \) Share of workforce in a region employed in sector \( s \)

\( K \) Land used in agricultural production

\( l \) Labour

\( \alpha_s \) Share of intermediates stemming from sector \( s \) used in production

\( w_{rs} \) Wages in region \( r \) and sector \( s \)

\( \nu_{rs} \) Wage differential in region \( r \) between agricultural sector and manufacturing sector \( s \)

\( E_{rs} \) Total expenditure in region \( r \) on goods of manufacturing sector \( s \)

\( g \) Mineral input into mining sector

\( \alpha_g \) Share of mineral input used in the mining sector

Table 4.1: Variables and Parameters in the Model

As a result, the utility function consists of two tiers. The upper tier defines expenditure allocation to the six sectors and is a variation of the
Cobb-Douglas form.

\[ U = (A - \bar{Y})^{1 - \sum \mu_s} \prod M_s^{\mu_s} \quad (4.1) \]

This utility formulation implies that a share \( 1 - \sum \mu_s \) of income above the subsistence level will be spent on the agricultural good, and \( \mu_s \) on the manufacturing sector \( s \). The lower tier expresses the consumer’s preference for variety within the subsectors. It defines \( M_s \) as a constant elasticity of substitution function and is equivalent to the subutility function in the simple VL model above, 3.2.

\[ M = \left[ \sum_{i=1}^{n} m_{is}^\rho \right]^{1/\rho} , \quad 0 < \rho < 1 \quad (4.2) \]

\( \rho \) is the indicator for the strength of a consumer’s love for variety for the different available products \( m_{is} \). \( n \) represents the number of firms and varieties available. As before, the elasticity of substitution can be calculated and equals

\[ \sigma = \frac{1}{1 - \rho} \]

A large \( \sigma \) (\( \rho \) is close to one) indicates that the differentiated manufacturing products are near perfect substitutes and consumers do not value variety highly. When \( \sigma \) is small \( \rho \) is close to zero and consumers display a great love for variety.

The budget constraint looks as follows:

\[ Y = p_a A + \sum_{s=1}^{5} G_s M_s \quad (4.3) \]

where \( G \) is a yet to be defined price index. In order to derive demand functions, it is necessary to maximize utility in two steps. The lower tier is addressed first to determine demand for a variety of a manufacturing good within any sector \( s \). Total expenditure on varieties within the sector is minimised subject to the constraint that overall utility equals \( M_s \).

\[ \min \sum_{i=1}^{n} m_{ip_i} \quad s.t. \quad \left[ \sum_{i=1}^{n} m_{ip_i}^\rho \right]^{1/\rho} = M_s \quad (4.4) \]
Setting up the Lagrangian function and differentiating for $m_i$ and $m_j$ respectively yields a price ratio of varieties $m_i$ and $m_j$

$$L = \sum p_i m_i + \lambda \left[ M_s - \left[ \sum m_i^\rho \right]^{\frac{1}{\rho}} \right]$$

$$\frac{p_i}{p_j} = \frac{m_i^{\rho-1}}{m_j^{\rho-1}}$$

From here, it is easy to find an expression for $m_i$

$$m_i = \left( \frac{p_j}{p_i} \right)^{\frac{1}{1-\rho}} m_j$$

Plugging this result back into the constraint yields

$$M_s = \left[ \sum_{i=1}^{n} \left( \frac{p_j}{p_i} \right)^{\frac{1}{1-\rho}} m_j \right]^{\frac{1}{\rho}}$$

Simplifying yields

$$M_s = \left[ \sum_{i=1}^{n} p_i^{\frac{\rho}{\rho-1}} \right]^{\frac{1}{\rho}} m_j p_j^{\frac{1}{\rho-1}}$$

Demand for an individual variety $m_j$ therefore turns out to be

$$m_j = M \frac{p_j^{\frac{1}{\rho-1}}}{\left[ \sum_{i=1}^{n} p_i^{\frac{\rho}{\rho-1}} \right]^{\frac{1}{\rho}}} \quad (4.5)$$

This is the compensated demand function. By rearranging terms, it can be rewritten to include a so-called price index $G$ as is shown in equations 3.7 and 3.8. $G$ thus equals

$$G = \left[ \int p_j^{\frac{\rho}{\rho-1}} d_j \right]^{\frac{\rho-1}{\rho}} \quad (4.6)$$

$G$ represents the minimum cost of obtaining one unit of the quantity index $M$. Not surprisingly, $G$ also reflects consumers’ desire for variety. If a larger number of varieties is available at a correspondingly lower price, $G$
will fall. The compensated demand function for $m_j$ can thus be rewritten by substituting the price index $G$ into 4.5.

$$m_j = M \left( \frac{p_j}{G} \right)^{\frac{1}{\rho - 1}}$$  

(4.7)

Or equivalent, if $\rho$ is replaced by the elasticity of substitution $\sigma$:

$$m_j = M \left( \frac{p_j}{G} \right)^{-\sigma}$$

Once the demand for a variety within a sector $s$ is known, the attention is turned to the upper tier of the utility maximisation problem. We maximise 4.1 subject to the budget constraint 4.3. Again, we set up the Lagrangian function:

$$LA = (A - \overline{Y})^{1 - \sum \mu_s} \prod M_s^{\mu_s} + \lambda(Y - p_A A - \sum G_s M_s)$$

Differentiating the Lagrangian function for $M_s$, $A$ and $\lambda$ and setting them to zero yields local maxima:

$$\frac{\partial LA}{\partial M_s} : \mu_s \frac{U}{M_s} = \lambda G_s$$

$$\frac{\partial LA}{\partial A} : (1 - \sum \mu_s) \frac{U}{A - \overline{Y}} = \lambda p_A$$

$$\frac{\partial LA}{\partial \lambda} : Y = p_A A + \sum G_s M_s$$

Take the sum of $G_s M_s$ over all industrial sectors plus the traditional sector to get

$$\sum_{s=1} G_s M_s + (A - \overline{Y}) p_A = \sum_{s=1} \mu_s + (1 - \sum_{s=1} \mu_s) \left( \frac{U}{\lambda} \right) = \frac{U}{\lambda}$$

In a next step, rewrite this expression and substitute $Y$ as derived above

$$Y - \overline{Y} p_A = \frac{U}{\lambda}$$

It is now possible to derive demand functions for the agricultural good and for manufacturing goods in each of the five sectors. Starting with the traditional sector and replacing $(\frac{U}{\lambda})$ in the local maximum above, I get

$$(A - \overline{Y}) p_A = (1 - \sum_{s=1} \mu_s)(Y - \overline{Y} p_A)$$
The demand for agricultural goods \( A \) then is
\[
A = \frac{(1 - \sum_{s=1}^{S} \mu_s)(Y - Yp_A)}{p_A} + \bar{Y}
\]  
(4.8)

The representative consumer spends a fraction of income above the minimal consumption requirement on the traditional good. Additionally, this minimal consumption requirement is spent on the traditional good as well.

The demand for industrial goods is derived in a similar manner. First, I derive demand for the composite of one industrial sector \( M_s \). Substitution equivalent to the procedure in the traditional sector yields
\[
M_s = \frac{\mu_s(Y - \bar{Y}p_A)}{G_s}
\]  
(4.9)

The consumer spends a fraction \( \mu_s \) of income over and above \( \bar{Y} \) on sector \( s \). However, we are interested in demand for a single variety produced by a firm within the sector. To achieve this, I replace \( M_s \) in Equation 4.7 by 4.9. Then, demand for a single variety becomes
\[
m_{sj} = \frac{\mu_s(Y - \bar{Y}p_A)}{G_s} \left( \frac{p_{sj}}{G} \right)^{\frac{1}{\rho-1}}
\]

Simplification yields the demand function for a single variety \( j \) in sector \( s \):
\[
m_{sj} = \mu_s(Y - \bar{Y}p_A)p_j^{\frac{1}{\rho-1}}G_s^{\frac{\rho}{\rho-1}}
\]  
(4.10)

Expressed in \( \sigma \), the elasticity of substitution, rather than in \( \rho \), demand can be rewritten as
\[
m_{sj} = \mu_s(Y - \bar{Y}p_A)p_j^{-\sigma}G_s^{\sigma-1}
\]  
(4.11)

Demand for a variety obviously depends on expenditure by consumers on the sector as established above. Within the sector, demand depends negatively on the price of the variety, and positively on the price index \( G_s \)—an increase in the price index indicates fewer varieties, hence less competition and therefore a larger share for one variety. It is also easily checked that the price elasticity of demand equals \( \sigma \) and is therefore constant.

As a last step in describing consumer behaviour, we can derive indirect utility \( V(Y, p_A, G_s) \) by substituting demand for \( A \) and \( M_s \) in the utility function 4.1.
\[
V = (1 - \sum_s \mu_s)^{1-\sum_s \mu_s} \prod_s \mu_s^{\mu_s}(Y - \bar{Y}p_A)p_A^{(1-\sum_s \mu)} \prod_s G_s^{-\mu}
\]  
(4.12)
The indirect utility function is useful when stressing once more the core feature of the particular formulation of utility and demand: consumers’ love for variety in the industrial sector. To formally re-establish this result, assume for now that prices $p_{sj}$ for all varieties $j = 1, \ldots, n$ in a sector $s$ are equal. Then the price index $G$ as expressed in 4.6 becomes $p_n\rho^{-1}/\rho$. If $n$ increases, the price index $G$ falls and utility increases. The extent of this preference for variety clearly depends on the parameter $\rho$—if it is close to one, a change in $n$ has little effect, if it is close to zero, then more varieties strongly affect $G$ and preference for variety is relatively larger.

Many Regions

So far, there has been little geography in this NEG-model. However, the purpose of this exercise is to analyse a country’s industrial development in a three-region setting. I will assume that consumers have identical preferences in the three regions, however one important adaption is necessary nonetheless: both agricultural and industrial goods will be tradable, allowing countries to import or export them corresponding to their specialisation in production.

For the sake of simplicity, it is generally assumed that trade in the agricultural good is costless to keep this sector as simple as possible. In the industrial sectors on the other hand trade is not free but generates transport costs of the iceberg-form. The NEG owes this formulation to von Thünen who introduced the concept already in the 19th century. What it assumes is that a fraction $1 - 1/T$ of the transported good melts on the way to its destination, so that only $1/T$ of the good arrives. This formulation has two major advantages: firstly, it is not necessary to formulate a separate transport sector. Secondly, firms will charge the same mill price to domestic and foreign consumers.

To see why it is necessary to look at firm behaviour and their profit-maximising price setting. For now, it suffices to understand the basic concept of iceberg transport costs. Assume that a manufactured good is produced in the North-region $N$. When consumed in the region, there are no transport costs, so the price of the good in the North will be $p_N$. Yet, when it is exported to the South-region $S$, only a fraction $1/T_{NS}$ of the good arrives. $T_{NS}$ stands for the amount of the manufactured good that has to be shipped for one unit to arrive in $S$. In terms of prices, a mill price of $p_N$ for one unit
implies a delivery price of \( p_N T_{NS} = p_{NS} \) in the South. Note also that \( T_{NN} \) equals one.

Transport costs will impact on the price indices in various regions and sectors, so they have to be reformulated. There are three regions \( R = 1, 2, 3 \) in this formulation. Moreover, it is necessary to assume that prices for individual varieties within a sector are equal, which is indeed the case here and will be shown later. Then, the price index in sector \( s \) of region 1 becomes

\[
G_{1s} = \left[ \sum_{r=1}^{3} n_{rs} (p_{rs} T_{1r})^{\rho} \right]^{\rho-1} \rho \tag{4.13}
\]

and might vary over locations. If a large proportion of goods are produced locally, the price index will be relatively low, in the opposite case (one might think of such a region as periphery) it will be relatively higher. Due to the nature of transport costs, demand has to be reformulated, and total sales of a variety have to include \( T \), the amount of the good lost in transport. So total sales of a single variety \( q \) in location 1 are

\[
q_1 = \mu_s \sum_{r=1}^{3} (Y_r - Y_r p_A) (p_j T_{1r})^{-\sigma} G_{s}^{\sigma-1} T_{1r} \tag{4.14}
\]

They depend on total income in all regions available for the consumption of manufacturing goods, consumers' preferences with regard to the different manufacturing sectors, the variety's price and transport costs. With regards to the demand elasticity, as above it is constant and equal to \( \sigma \). Note that it does not depend on transport costs either. As indicated above, this will be convenient at a later stage when we look at the price setting of firms.

**Producer Behaviour**

With consumer behaviour and demand established, we can look at the behaviour of firms. In the traditional sector, there is perfect competition, trade between regions is costless and its output will be used as numéraire. In the manufacturing sectors, the market form is monopolistic competition, as introduced by Dixit and Stiglitz (1977).

Turning our attention to the agricultural sector first, we assume a production function that is increasing in the input labour, yet with diminishing returns. Following Fujita et al. (2001, 257) it takes the following form:

\[
q_A = \frac{K}{\gamma} \left( \frac{1 - \sum s \delta_s}{K} \right)^\gamma \tag{4.15}
\]

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$K$ can be thought of as an agriculture-specific input into production such as land. The other input factor of course is labour. A share of $(1 - \sum_s \delta_s)$ of the total labour force in a region (which is set to one for simplicity) work in the agricultural sector. The rest of workers are distributed over the five manufacturing sectors, with sector $s$ employing $\delta_s$ workers.

The marginal product of labour in agriculture is positive yet diminishing. If a region industrialises—in other words if workers are drawn out of agriculture and move into manufacturing—then the marginal product increases and so does the agricultural wage. Since workers move freely between sectors within a region, the industrialisation process also increases the equilibrium manufacturing wage. This is a core feature of the model which is primely responsible for the setting up of new industries in the periphery.

Since the agricultural good acts as numéraire, the agricultural wage will equal the marginal product of labour in the sector. Formally,

$$w_A = \left(\frac{1 - \sum_s \delta_s}{K}\right)^{\gamma-1} \quad (4.16)$$

In each manufacturing sector, firms will produce with increasing returns to scale since there is a fixed-cost component involved in the production process. Because of this, each firm will produce one variety only. Because of consumers’ love for variety, we also know that each variety will be produced by only one firm. The number of firms in a sector will thus equal the number of varieties on offer. The inputs used are labour and intermediate goods that come from the five manufacturing sectors. The formulation is thus equivalent to the simple VL model—equation 3.17. The production function for a typical firm in sector $s$ then equals

$$F + cq_s = (1 - \sum \alpha_s) \sum_s \alpha_s^{-1} \prod_s \alpha_s^{-\alpha_s} l^{\alpha_1} \prod_s M_s^{\alpha_s} \quad (4.17)$$

To put the production function in this manner immediately points to one of its crucial attributes: the inputs, labour and intermediate goods, are used in both the fixed and the variable component of production. In other words, to set up the factory, the firm incurs fixed costs $F$, of which a proportion $1 - \sum \alpha_s$ is spent on workers, and the rest (shares $\alpha_s$) on inputs from the different manufacturing sectors. The same proportionality applies to variable costs. On the right hand side, the first two terms are constants that ensure a simple cost function (see below). $l$ represents the labour input in production, $M_s$ the intermediate input from a particular sector $s$. 

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Ms again represents a manufacturing goods composite for a specific sector and consists of a number of differentiated varieties. Firms display a love for variety in their input demand equivalent to consumer preferences. For the sake of simplicity, the formulation will be exactly equivalent. Reformulating, we get

\[
F + cq_s = \left(1 - \sum_s \alpha_s\right) \prod_s \alpha_s - \alpha_s l^{-1} - \sum_s \alpha_s \prod_r \left[\sum_r n_{rs} x_{rs}^\rho\right]^{\alpha_s \rho} \tag{4.18}
\]

The CES aggregator \(\left[\sum_r n_{rs} x_{rs}^\rho\right]^{1/\rho}\) shows how a firm uses differentiated inputs \(x\) stemming from sectors \(s = 1,..5\) and from all three regions. It has a preference for variety, and the strength of this preference depends on \(\rho\).

To arrive at profit maximization and price setting of an individual manufacturing firm, we need to know the cost function first. This involves some tedious algebra, but the result is a rather intuitive cost function. Starting with the production function 4.17 the Lagrangian function can be set up:

\[
LA = wl + \sum_s G_s M_s + \lambda \left[q - (1 - \sum_s \alpha_s) \prod_s \alpha_s - \alpha_s l^{-1} - \sum_s \alpha_s \prod_s M_s^{\alpha_s} \frac{1}{c} + \frac{F}{c}\right]
\]

Then

\[
\frac{\partial LA}{\partial l} : w = \lambda(1 - \sum_s \alpha_s) l^{-1} \sum_s \alpha_s - \sum_s \alpha_s \prod_s \alpha_s - \alpha_s l^{-1} - \sum_s \alpha_s \prod_s M_s^{\alpha_s} \frac{1}{c}
\]

\[
\frac{\partial LA}{\partial M_{s1}} : G_{s1} = \lambda \alpha_{s1} M_{s1}^{\alpha_{s1} - 1} \prod_s M_s^{\alpha_s} \left(1 - \sum_s \alpha_s\right) \prod_s \alpha_s - \alpha_s l^{-1} - \sum_s \alpha_s \prod_s M_s^{\alpha_s} \frac{1}{c}
\]

\[
\frac{\partial LA}{\partial M_{s2}} : G_{s2} = \lambda \alpha_{s2} M_{s2}^{\alpha_{s2} - 1} \prod_s M_s^{\alpha_s} \left(1 - \sum_s \alpha_s\right) \prod_s \alpha_s - \alpha_s l^{-1} - \sum_s \alpha_s \prod_s M_s^{\alpha_s} \frac{1}{c}
\]

\[
\frac{\partial LA}{\partial \lambda} : q = (1 - \sum_s \alpha_s) \prod s \alpha_s - \alpha_s l^{-1} - \sum_s \alpha_s \prod s M_s^{\alpha_s} \frac{1}{c} - \frac{F}{c}
\]

From this, by division we arrive at much simplified price ratios:

\[
w l \alpha_{s1} = G_{s1} M_{s1} \left(1 - \sum_s \alpha_s\right)
\]

\[
G_{s2} M_{s2} \alpha_{s1} = G_{s1} M_{s1} \alpha_{s2}
\]

If we want to express input demand for \(l\) and \(M_s\) in terms of output and prices, these ratios have to be plugged back into the production function.
Input demand for labour $l$ can be derived by inserting $M_s = \frac{wl\alpha_s}{G_s(1-\sum\alpha_s)}$ in the production function. The relevant term $\prod_s M_s^{\alpha_s}$ becomes

$$\prod_s M_s^{\alpha_s} = (wl)\sum\alpha_s(1 - \sum\alpha_s)^{-\sum\alpha_s} \prod_s \left(\frac{\alpha_s}{G_s}\right)^{\alpha_s}$$

and $l$ turns out to be

$$l = \frac{(F + cq)(1 - \sum\alpha_s) \prod G_s^{\alpha_s}}{w\sum\alpha_s}$$

The equivalent procedure produces input demand for the composite $M_s$ of one industrial sector. I insert $l = G_s M_s(1 - \sum\alpha_s)/(w\alpha s)$ into the production function which turns out to be

$$F + cq = \prod \alpha_s^{-\alpha_s} \frac{2^{-5}}{\sum\alpha_s} \prod M_s^{\alpha_s} M_s^{1 - \sum\alpha_s} G_s^{1 - \sum\alpha_s}$$

With a further substitution, this expression can be greatly simplified. The product of manufacturing composites $\prod M_s^{\alpha_s}$ can be split up, extracting $M_s^{\alpha_s}$ and replacing the remaining sectoral composites $s2$ to $s5$ by $M_s^{\alpha_s} = G_s M_s(1 - \sum\alpha_s)/(G_s \alpha s)$ and so forth. Then

$$F + cq = \prod \alpha_s^{-\alpha_s} \alpha_s^{\sum\alpha_s - 1} G_s^{1 - \sum\alpha_s} \prod G_s^{-\alpha_s} M_s^{1 - \sum\alpha_s}$$

Simplication yields

$$F + cq = \alpha_s^{-1} M_s w^{\sum\alpha_s - 1} G_s^{1 - \sum\alpha_s} \prod G_s^{-\alpha_s}$$

From this, I derive an expression for $M_s$

$$M_s = \frac{(F + cq)\alpha_s}{w\sum\alpha_s - 1 G_s^{1 - \sum\alpha_s} \prod 2^{-5} G_s^{-\alpha_s}}$$

Lastly, factor demand is inserted into the total cost function $TC = wl + \sum G_s M_s$. Then, $TC$ can be expanded to

$$TC = w\left(\frac{(F + cq)(1 - \sum\alpha_s) \prod G_s^{\alpha_s}}{w\sum\alpha_s}\right) + \sum G_s \frac{(F + cq)\alpha_s}{w\sum\alpha_s - 1 G_s^{1 - \sum\alpha_s} \prod 2^{-5} G_s^{-\alpha_s}}$$

Simplify to get

$$TC = (F + cq) w^{-\sum\alpha_s} \left[(1 - \sum\alpha_s) \prod G_s^{\alpha_s} + \sum G_s \frac{\alpha_s}{G_s^{1 - \sum\alpha_s} \prod 2^{-5} G_s^{-\alpha_s}}\right]$$
and finally

\[ TC = (F + cq) \left[ w^{1-\sum \alpha_s} \prod G_s^{\alpha_s} \right] \]  

(4.19)

This simple and intuitive cost function shows how production costs (both the fixed cost component and variable costs) depend on wages and price indices of intermediate inputs, impacting according to the share of expenditure that goes towards their use.

We can now formulate the profit maximization problem of a manufacturing firm.

\[ \Pi = pq - (F + cq) \left[ w^{1-\sum \alpha_s} \prod G_s^{\alpha_s} \right] \]  

(4.20)

where total demand \( q \) is taken from equation 4.14. The firm will set production so that marginal revenue equals marginal costs or \( p(1 + \frac{1}{\epsilon}) = MC \).

The monopolistic competition formulation implies that the firm—producing a distinct variety—acts as a local monopolist and does not consider its competitor’s reaction. It thus takes the price index \( G \) as given, which in turn implies that the elasticity of demand is \( \sigma \).

Algebraically,

\[ p(1 + (\frac{1}{\sigma})) = c \left[ w^{1-\sum \alpha_s} \prod G_s^{\alpha_s} \right] \]

Replace \( \sigma \) by \( \rho \) and reformulate to arrive at the price set by the producer of a variety:

\[ p = \left[ w^{1-\sum \alpha_s} \prod G_s^{\alpha_s} \right] \frac{c}{\rho} \]  

(4.21)

Prices are set at a constant mark-up \( \frac{c}{\rho} \) over marginal costs. On the other hand, entry into the market is free, and profits will be driven down to zero accordingly. We use the zero-profit condition to determine the size of an individual firm.

Setting \( \Pi = 0 \) and using 4.21 we get

\[ \left[ w^{1-\sum \alpha_s} \prod G_s^{\alpha_s} \right] \frac{c}{\rho} q = (F + cq) \left[ w^{1-\sum \alpha_s} \prod G_s^{\alpha_s} \right] \]

Simplifying yields equilibrium output of a firm

\[ q^* = \frac{F \rho}{(1-\rho)c} \quad \text{or} \quad q^* = \frac{F(\sigma - 1)}{c} \]  

(4.22)
Firm size thus depends on the cost parameters (positively on fixed costs, negatively on variable costs) and on the preference for variety of consumers. The higher the representative consumer’s love for variety as expressed in $\sigma$ or in $\rho$, the smaller the output of an individual firm—more firms will offer more varieties in smaller quantities. Note also what does not affect firm size: the overall size of the market. If total demand increases, it will not change supply behaviour of a single firm. What it does cause however is the entry of new firms—a larger market then leads to a larger choice of varieties and thereby reduces the price index $G$ in the market. More formally, to express the number of firms as a function of market size, equate the total wage bill in a sector $s$ with the share of total revenue in that sector that goes to wages:

$$\delta_s w = (1 - \sum \alpha_s) npq$$

Thus, the equilibrium number of firms becomes

$$n^* = \frac{\delta_s w}{(1 - \sum \alpha_s) pq} \quad (4.23)$$

Before writing down the short run equilibrium, we have to reformulate total demand for goods because we now know that firms also demand industrial products as part of their input demand. Total demand from consumers including the amount of goods lost in transport was recorded in 4.14. Now demand from firms has to be added. Due to the formulation of their production function 4.17 they spend a fixed proportion $\alpha_s$ of their total costs on products stemming from sector $s$. Within sectoral demand, they have a preference for variety equivalent to consumer behaviour, depending on the parameter $\rho$.

So total demand for a variety $i$ in sector $s$ can be written as

$$q_{si} = \sum_r E_{rs} \left[ p_i^{-\sigma} \left( \frac{G}{T} \right)^{\sigma - 1} \right] \quad (4.24)$$

where expenditure on sector $s$, $E_s$ in one particular region is defined as

$$E_s = \mu_s (Y - \bar{Y} p_A) + \sum_z \alpha_{zs} npq \quad (4.25)$$

Consumers spend a share $\mu_s$ of income above the minimum agricultural good requirement $\bar{Y}$ on the sector, and firms from sectors $z = 1...5$ spend an according share $\alpha_{zs}$ of their total sales—reflecting the input-output linkages between sectors.
Short Run Equilibrium

We are now in a position to write down the short run equilibrium of the model. It is expressed in the price indices, the so-called wage equations, expenditure and income. In order to simplify notation, a number of normalizations are introduced.

\[ p_A = 1 \]
\[ c = \rho \]
\[ q^* = \frac{1}{1 - \sum \alpha_s} \]

We start by defining expenditure \( E_{rs} \) in a region \( r \) on products of the manufacturing sector \( s \). Use the normalisations above and transform 4.25 accordingly to get

\[ E_{rs} = \mu_s(Y_r - \bar{Y}_r) + \sum_z \alpha_{zs} \frac{\delta_s w}{(1 - \sum \alpha_s)} \]  
(4.26)

The first term on the right hand side is consumption expenditure on a sector (dependent on \( \mu_s \)), the second term intermediate demand from firms (dependent on \( \alpha_{zs} \)).

Income \( Y_r \) in a region \( r \) comes from workers’ wages in manufacturing and from income in the agricultural sector. Thus

\[ Y_r = \sum_s w_s \delta_s + \frac{K}{\gamma} \left( \frac{1 - \sum_s \delta_s}{K} \right)^\gamma \]  
(4.27)

Turning to the price indices, we reformulate 4.13 to get the price index in region \( r_1 \) and sector \( s \)

\[ G_{r_1 s}^{1-\sigma} = \sum_r n_{rs} (p_{rs} T_{r_1 r})^{1-\sigma} \]

After the normalisations introduced above, the price \( p \) as in 4.21 and the number of firms \( n \) (see equation 4.23) can be simplified and rewritten as

\[ p = w^{1-\sum \alpha_s} \prod G_s^{\alpha_s} \]
\[ n = \frac{\delta_s w}{(1 - \sum \alpha_s) pq^*} \]
\[ n = \frac{\delta_s w}{w^{1-\sum \alpha_s} \prod G_s^{\alpha_s}} \]
Plug these into the price index to get

\[
G_{r_1s}^{1-\sigma} = \sum_r \frac{\delta_s w}{w^{1-\sum \alpha_s}} \prod G^{\alpha_s} (\prod G^{\alpha_s})^{1-\sigma} T^{1-\sigma}
\]

Simplification yields the final version of the price index for region \( r_1 \) and its sector \( s \):

\[
G_{r_1s}^{1-\sigma} = \sum_r \delta_s w^{1-\sigma(1-\sum \alpha_s)} \prod G^{-\alpha s} T^{1-\sigma}
\]  
(4.28)  

Equation 4.28 shows that the price index in a region first of all depends on transport costs to the region—the higher they are, the higher the price index will be. Also, this implies that a remote region that imports a larger share of its manufactured goods will face higher price indices. The term \( \delta_s Lw \) is an indicator of the market size of respective regions. Lastly, the price index depends on prices of individual varieties, and since prices are set as a mark-up over marginal costs, wages and input prices enter the equation as well.

Lastly, we define the so-called manufacturing wage equations. Manufacturing wages are crucial since they will determine the dynamics of the model. Yet they are intuitively hard to grasp since there is no closed form expression for them. We arrive at them by using what Baldwin et al. (2003, 19) call the “market clearing conditions”. From 4.22 we know the output a firm produces at which it can exactly recover its fixed costs and so has zero profits. From demand by consumers and firms, we thus know that this output level \( q^* \) must equal

\[
q^* = \sum_r E_{rs} \left[ p_i^{-\sigma} \left( \frac{G}{T} \right)^{\sigma-1} \right]
\]

Rearrange to bring the price on the left hand side. This price \( p_i \) is the break even price that a firm must charge in order to achieve zero profits. If it sets its price at a lower level, it will go out of business because it operates at a loss, if it sets its price at a higher level, competitors will enter the market and drive the price down.

\[
p_i^\sigma = \frac{1}{q^*} \sum_r E_{rs} \left[ \left( \frac{G}{T} \right)^{\sigma-1} \right]
\]

Lastly, price setting is linked to manufacturing wages due to mark-up pricing. Thus, we can indirectly define the manufacturing wage at which firms
achieve zero profits, given income, price indices and transport costs in all locations.

\[
\left[w^{1-\sum\alpha_s} \prod G_s^\alpha_s \right]^\sigma = \frac{1}{1-\sum\alpha_s} \sum_r E_{rs} \left[\left(\frac{G}{T}\right)^{\sigma-1}\right]
\]  

(4.29)

This is the so-called wage equation. We can use it plus the price indices to illustrate the main forces at work in the model: backward and forward linkages and the market-crowding or competition effect. The question to be asked is whether any given short run equilibrium with resulting income, expenditure, price indices and wages in all regions represents a stable equilibrium. In order to find out, assume that a small number of workers move into manufacturing in one region and \( \delta_s \) changes accordingly—will that trigger further specialisation in manufacturing (in other words: will manufacturing wages increase further) because of linkage effects? Or will the competition effect prevail and thus prevent further specialisation?

- **Backward Linkages:** Backward linkages stem from the increasing size of the local market that results from more manufacturing production in the region. This increase works through both input demand by manufacturing firms and through additional final demand by workers and is relevant because of the existence of positive transport costs. In equations 4.26 to 4.29 the backward linkage shows up in expenditure on manufacturing goods \( E \). If \( \delta_s \) increases, so does expenditure in the region. This shifts up demand curves for local producers and leads to an increase in wages.

- **Forward Linkages:** Forward linkages represent the cost reduction for downstream producers that is associated with the local availability of intermediate inputs. In the model, this effect relies on the preference for variety of firms: if more varieties of an input good are available locally, the price index of that input will decrease. When \( \delta_s \) increases in 4.28, the regional price index in the respective sector falls. In terms of wages, this reduction in input costs tends to increase the equilibrium wage further, as can be seen in \( G_s \) on the left hand side of the wage equation.

- **Competition Effect:** The competition effect provides an opposing tendency. The increase in \( \delta_s \) that reduces the price index also implies that
the demand for each firm’s variety shifts down. This will tend to reduce the equilibrium wage and is represented in the model in $G_s$ on the right hand side of the wage equation. Lastly, an increase in $\delta_s$ simultaneously leads to a decrease in agricultural employment $(1 - \sum_s \delta_s)$. This raises the marginal product of agricultural labour and thus agricultural wages. Competition for workers thus puts another break on industrialisation.

Wages are the reward for the mobile factor, labour. The short run equilibrium as defined above in equations 4.26 to 4.29 yields a wage rate in each of the manufacturing sectors. They might and will in all likelihood differ, depending on the respective forward and backward linkages and the competition effect. These differing wages in the manufacturing sectors, or, more precisely, the wage differential between the agricultural sector and manufacturing sectors, then determine the sectoral movement of workers in the long run and thus the dynamics of the model. If wages are higher in manufacturing, workers will move there and the region amasses industrial production. In the opposite case, they will leave the modern sector and the region deindustrialises. Note that firms always operate at zero profits, which means that they immediately move into or out of a sector according to the profit signal. Workers on the other hand do not move in the short run but only switch occupation in the long run equilibrium.

**On Dynamics**

The wage differentials in the short run trigger movements of workers to those industries that offer higher wages. In the long run equilibrium, wages will equalise across sectors. Note that they do not equalise across regions since labour is not interregionally mobile. This is one of the core differences between the core-periphery and the vertical linkages driven NEG models.

Therefore it is important to define the mobility equations that describe the movement of workers. The incentive to move is provided by the wage differential between the traditional sector and a particular manufacturing sector. We define the wage differential in region $r$ and sector $s$ $v_{rs}$ as

$$v_{rs} = w_{rs} - \left( \frac{1 - \sum_s \delta_s}{K_r} \right)^{\gamma^{-1}}$$  \hspace{1cm} (4.30)

If $v_{rs}$ is positive, agricultural workers will leave their sector and move into manufacturing sector $s$ thereby increasing its size and triggering above men-
tioned adjustments. The mobility equation is then defined as

$$\dot{\delta}_{rs} = v_{rs} \gamma (\delta_{rs} (1 - \delta_{rs})) \quad (4.31)$$

The change over time in a sector's workforce depends on its wage differential to the agricultural sector. The formulation in 4.31 ensures that the movement stops in the corner solutions—if all workers are working in sector $s$, $(1 - \delta_{rs})$ is zero and movement stops, if the sector does not exist ($\delta_{rs} = 0$), the same applies.

Long run interior equilibria can be established with simulation analysis—results will depend on parameter values chosen. Both will be described in detail in the sections below. Before that, one last building block of the model has to be introduced however—the exogenous growth process. We are not so much interested in the causes of growth in this model, but in its effects on the sectoral composition of output. Therefore, the formulation will be very simple: technological progress steadily increases the productivity of labour. Following Fujita et al. (2001, 264), labour is denoted in efficiency units $L$. Thus $L\delta_{rs}$ is the number of efficiency units operating in country $r$ and sector $s$, and $w_{rs}$ is the wage per efficiency unit. So if the economy grows thanks to an increase in $L$, workers' income through wages increase.

The short run equilibrium has to be reformulated to accommodate for these changes.

$$E_{rs} = \mu_s (Y_r - \bar{Y}_r) + \sum_z \alpha_{zs} \frac{L\delta_s w}{(1 - \sum \alpha_s)} \quad (4.32)$$

$$Y_r = \sum_s w_s L\delta_s + L \frac{K}{\gamma} \left( \frac{1 - \sum_s \delta_s}{K} \right)^\gamma \quad (4.33)$$

$$G_{r_1 s}^{1 - \sigma} = \sum_r L\delta_s w^{1 - \sigma(1 - \sum \alpha_s)} \prod_s G_{rs}^{-\alpha_s} T_{r_1 r}^{1 - \sigma} \quad (4.34)$$

$$\left[ w^{1 - \sum \alpha_s} \prod_s G_{rs}^{\alpha_s} \right]^{\sigma} = \frac{1}{1 - \sum \alpha_s} \sum_r E_{rs} \left[ \frac{G}{T} \right]^{\alpha - 1} \quad (4.35)$$

The growth process is relevant because of the particular form of utility and demand: once income exceeds $\bar{Y}$, a fixed proportion is spent on manufacturing goods. In total, this implies that the demand for manufacturing goods grows faster than the demand for agricultural goods. These shifts are responsible for sectoral changes: additional manufacturing production will
happen in regions where it is most profitable, either due to linkage effects or due to lower wage costs. The development of this growth-induced process over time and the various forces that drive these results will be analysed below.

4.1.3 The Mining Sector

The first of the non-traditional sectors will represent the mining sector. In addition to inputs from all other sectors, it also uses a mineral input in production. I call it $g$. A constant share $\alpha_g$ of total costs is spent on the mineral input in the mining sector. In all other sectors, $\alpha_g$ equals zero. From this follows the production function\(^1\) for the mining sector:

$$ F + cg = l^{1-\sum \alpha_s - \alpha_g} g^{\alpha_g} \prod_s M_s^{\alpha_s} $$  (4.36)

I further assume that the price of the mineral input is exogenously given. This simplifies my analysis considerably and is not an unreasonable assumption—given the size of South Africa’s economy and the global market for natural resources. It is denoted $p_g$. Total costs in the minerals sector in the region profiting from being endowed with it thus are

$$ TC = (F + cg) \left[ w^{1-\sum \alpha_s - \alpha_g} p_g^{\alpha_g} \prod G_s^{\alpha_s} \right] $$  (4.37)

In the other regions, the mineral input has to be imported. Its import bears iceberg transport costs of the magnitude $T$, so the input cost accordingly is $p_g T$.

Just as in the rest of the sectors, firms in the mining sector set prices as a mark-up over marginal costs, which—thanks to the normalisations introduced above—disappears. The price set in the mining sector $p_{min}$ therefore is

$$ p_{min} = w^{1-\sum \alpha_s - \alpha_g} p_g^{\alpha_g} \prod G_s^{\alpha_s} $$  (4.38)

Returning to the short run equilibrium, the price indices and the wage equations have to be adapted to cater for the special role of the mining sector. The price index of the sector as described in 4.34 changes to

$$ G_{r1}^{1-\sigma} = \sum_r L \delta_s p_g^{\alpha_g} w^{1-\sigma(1-\sum \alpha_s - \alpha_g)} \prod_s G_s^{\alpha_s} T^{1-\sigma} $$  (4.39)

\(^1\)This is a simplified version, omitting the constants
in the region with the mineral endowment. In the other regions, the price of the mining input enters as \((T_p g)^{-\alpha_g}\). The wage equation is adapted accordingly. Again, in the mineral-rich region, I replace the price set by firms in the mining sector in the wage equation to get

\[
[p_g^\alpha_g w^{1-\sum \alpha_s - \alpha_g} \prod G_s^\alpha_s]^{\sigma} = \frac{1}{\sum \alpha_s} \sum_r E_{rs} \left(\frac{G}{T}\right)^{\sigma-1}
\]  

(4.40)

In the other regions, transport costs for the mineral input will be considered on the left hand side again. Now that the special features of the minerals sector are established, numerical simulation will shed more light on the model and its characteristics. The focus of attention will lie on the role of the mineral endowment and its consequences on sectoral growth in the three regions.

4.2 Simulation Results

In this section, I present results for two simulations. The first uses a very simplified input-output matrix to illustrate the features of the model, particularly the effect of the mineral endowment on sectoral industrial development and wages in the respective regions. In the second simulation, I attempt to use a more realistic input-output matrix that more accurately describes the South African case.

4.2.1 Three Regions, Five Industries: a Simple Case

The simulation presented here uses parameter values that are standard in the literature and describe a situation of low transport costs where a core-periphery outcome is expected. They are described in more detail in the appendix.

The crucial aspect that needs to be highlighted and that differs from work done before is the input-output matrix that links the five manufacturing sectors. Table 4.2 displays all values of \(\alpha_s\) for industry 1 in column 1, for industry 2 in column 2 and so forth. So each column presents the total of intermediate inputs from other modern sectors for the respective sector. Sector 1, the mining sector, additionally uses the mineral good as input. \(\alpha_g\), the share of the mineral resource in the production of the mining sector is set to \(\alpha_g = 0.5\) to represent its dominance in this sector.
Table 4.2: Input-Output Matrix

<table>
<thead>
<tr>
<th></th>
<th>MEC</th>
<th>Non-MEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_1)</td>
<td>0.11</td>
<td>0.12</td>
</tr>
<tr>
<td>(a_2)</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>(a_3)</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>(a_4)</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>(a_5)</td>
<td>0.01</td>
<td>0.02</td>
</tr>
</tbody>
</table>

The matrix is constructed so that industries 1 and 2 are heavily linked. So the second industry represents those activities that use the mining industry’s output heavily and also provide significant inputs to the mining industry. In terms of the South African economy that provides the underlying reference for this simulation, the second sector can be identified as those manufacturing sectors that together with mining itself make up the Minerals-Energy Complex; that is base chemicals, petroleum products, and plastics, non-metallic mineral products, base metals, and electricity. Sectors 3 to 5 on the other hand are weakly linked to the resource-intensive core of the economy—both in terms of forward and backward linkages. However, they display strong linkages among themselves. They represent non-MEC manufacturing sectors in the simulation.

The simulation is done with Mathematica. For an initial value of \(L = 1\), the short run equilibrium price indices, wages, expenditure and income are calculated. From this, the wage differentials \(V_s\) are gathered in all three regions and their respective sectors. Workers then move according to the law of motion as defined in 4.31. A long-run equilibrium is reached once they have no more incentive to leave their sector. In the graphic representations below, I display subsequent long run equilibria for increasing values of \(L\)—thus a simulation of the growth process.

Region 1 represents the core region, region 2 profits from the minerals endowment, and region 3 is the periphery. In the starting position, the core region specializes in activities 3 to 5, the second region in those activities close to the mineral endowment, and the third region is the periphery, accounting for no modern production at all. When the economy starts to grow, the first two regions start to amass sectoral activity according to the initial sectoral distribution. Country 3 remains peripheral. At a critical
level however, incentives change and it becomes attractive for some manufacturing firms to leave the core region and set up shop in the periphery. The simulation results show that—despite a small hike in region 2—region 3 establishes itself as a manufacturing region (see Graph 4.1). The mineral-rich region keeps its competitive advantage in the first two sectors, but the other industries are now more equally divided between region 1 and region 3. Firms in these sectors do not set up shop in region 2.

What are the driving forces of these results? The linkage structure imposed on the model clearly plays a decisive role. Strong linkages between industries 1 and 2 explain why they are tied in the second region. These linkages also explain the closely correlated development of industries 3 to 5.
However, the input-output matrix cannot explain why the growth process triggers industrialisation in region 3—without any manufacturing activity and thus without any benefits accruing from linkage effects—and not in region 2. The reason for this are wage costs (see Graph 4.2). Not surprisingly, the core is the high wage region initially. Higher economic activity drives up manufacturing wages through the channels discussed above. The growth process further increases manufacturing wages in the core, until, at the break point, the wage costs are so high that it pays for some firms to establish themselves in the periphery.

Between the two peripheral regions, the mineral-rich region has some manufacturing activity—thus there will be linkage-related advantages for firms that emerge there. However, as can be seen in Graph 4.2, the presence of firms in sectors 1 and 2 implies that wages in the region are higher than in region 3 that is fully specialized in the traditional sector. The wage differential between them is large enough to overshadow linkage advantages and therefore manufacturing activity in sectors 3 to 5 emerges in region 3, the low-wage region.

Once some firms have made that step, the region profits from local linkage effects as well and quickly industrialises. Wages will rise accordingly and the country soon reaches the level of the industrialised core in terms of wages. Region 2 grows continuously in terms of wages, but fails to attract a broader set of industries: its development is clearly tied to the mineral
endowment. Albeit an advantage initially, it puts an upper limit on sectoral growth and eventually also on wages and thus welfare in the region, because it prevents a diversification process.

4.2.2 Three Regions, Ten Industries: South Africa

The results obtained above stem—to a good extent—from the peculiar form of the input-output matrix chosen above. In order to reestablish the results with an industry structure closer to what we observe, I run a simulation with 3 regions and 10 industries that uses an input-output matrix modelled on South Africa’s industrial structure.

The derivation of the matrix is explained in the Appendix. The outcome is a 10x10 matrix as presented in Table 4.3. Sector 1, mining and quarrying, represents the mining sector from above and uses the mineral as additional input again. Apart from this, parameter values are very similar to those chosen above (see Appendix), that is we are in a setting of low transport costs. The three countries are identical in all respects except the minerals endowment. Region 2 has access to the mineral without transport costs, the other regions have to import it and thus have to bear additional transport costs.

With regards to the input-output matrix, it is clear the sector ‘petroleum products, chemicals, rubber and plastic’ is the one that is most closely linked to the mining sector. A share of 0.145 of its total production costs is spent on inputs stemming from the minerals sector. This corresponds to the empirical literature that identifies this sector as crucial component of the MEC. The other manufacturing sectors that are usually subsumed under the label MEC are basic metals, a sub-group of the sector-grouping ‘metals, metal products, machinery and equipment’ in the table. However, its backward linkage to the mining sector (0.086), albeit larger than that of the rest of sectors, is less important than the linkage of the chemicals sector.

A major difference to the hypothetical 5 industries-case presented above is that ‘the rest of the economy’, that is the sectors that are not considered part of the MEC, are not in a systematic way closely linked with each other. This was an artefact above and will be visible in the results below. One does observe stronger linkages between certain sectors however, notably the importance of metal products in the transport equipment sector and, to a lesser extent, in the electrical machinery sector.
Table 4.3: Input-Output Matrix

Results of the simulation show a similar pattern of sectoral industrial development. However it differs from the simulation above in important aspects. Turning the attention to the development of the 10 sectors in the three regions first (see Figure 4.3), it is obvious that the growth process is more continuous for the majority of sectors in the three regions than before. The one major exception is the ‘Chem’ sector, that is initially located in the core region and relocates to the minerals-rich region once a certain growth level is attained. This relocation is triggered by the wage differential becoming big enough for some firms to relocate, and happens in this particular sector because of the strong linkages to the minerals sector that is overwhelmingly located in region 2. These changes lead to a familiar pattern of sectoral development in the region: mining and the closely related sectors dominate the economy. Given the input output structure of South Africa, this applies for the petroleum products, chemicals and plastics sector. The rest of the economy virtually stagnates, there is no significant growth in any of the other sectors despite a continuous increase in world wide demand for manufacturing products.
This additional demand is satisfied by firms that emerge either in the core region, where they build on existing advantages of agglomeration—expressed in the form of linkages that render intermediate inputs cheaper in this model, or in region 3, the periphery. The core, while loosing activity in the ‘Chem’ sector, grows strongly in the metal products sector, and this growth pulls along transport equipment and electrical machinery, that are both strongly
linked to metal products. Less intuitive is the strong performance of the food sector in the core.

With regards to the periphery, its growth is concentrated predominantly in the wood and paper and in the textiles industry. Those two sectors exceed their counterparts in the mineral-rich region in size. The periphery unsurprisingly does not engage in mining or closely linked industries at all. Intuitively plausible as well, there is no metal products and transport equipment production in it either.

The major difference to the first simulation is that region 3 does not fully industrialise and thus overtake the minerals-rich region. The reason for this difference lies in the more realistic input-output matrix. Inter-industry linkages in the ‘non-mineral’ sectors (sectors 3-5 above) are nowhere near as strong and therefore do not trigger a rapid industrialisation process in all of them once activity emerges in one of them.

This difference is also visible in the development of wages. In the early stages of the growth process, the wage gap widens between the core region and the two peripheral regions. The relocation of the ‘Chem’ sector then strongly increases economic activity and wages in the minerals-rich region, bringing it to a wage level much closer to the core region. Region 3 remains a low-wage region. As a result of the wage difference between regions 2 and
3 however, most sectors outside mining and chemicals are more important in the periphery than region 2.

So while the second simulation does not replicate the strong welfare results of the first exercise (an overtaking of the minerals-rich region by the peripheral region in terms of wages), it does retain the sectoral implications of input-output linkages and the mineral endowment: it is the sector closely linked to mining that will relocate to the mineral-rich region, and this increased economic activity in one sector, by driving up wages in the economy as a whole, dampens economic activity in a host of other sectors. Given the parameter values chosen here, these results are strongest for the sectors ‘wood and paper; publishing and printing’, ‘textiles, clothing and leather’ and ‘electrical machinery, radio, TV, and instruments’, that all develop more dynamically in the peripheral region.

The core region retains its high wages mostly through dominance in ‘metals, metal products, machinery and equipment’, ‘transport equipment’ and, certainly less intuitive, ‘food, beverages and tobacco’.

At this point, it is important and necessary to stress the limits of this analysis. Many, if not most, important factors influencing sectoral development have been abstracted from, not least additional production inputs in other sectors (for example in the wood and paper sector). Regarding the simulation, it is therefore apparent that one cannot speak of an empirical application proper of an economic geography model. I therefore second Forslid et al. (2002) who called simulations of a similar kind “theory with numbers” (Forslid et al., 2002, 281). What the simulation does provide though is a theoretical argument that justifies further investigation into the importance of linkages and thus ‘real’ empirical work.

In the next chapter, I will attempt to do just this and provide some empirical evidence for the importance of linkages in the context of South African industrial development and the growth performance of its various sectors.