Chapter 5

Conclusion

In this work we studied the stochastic single-period, single-item inventory control & pricing problem under spectral measures of risk. The class of spectral risk measures is general in the sense that it can express risk-averse, risk-neutral and risk-seeking risk preferences. It can cover the well-known CVaR$_\alpha$, as well as mean-deviation criteria or continuous risk functions; the power and exponential risk spectra are special cases of spectral risk measures. Using this class of risk measures allows us to generalize structural results obtained so far in the literature.

We divided the problem analysis into two main parts: first we derived optimality conditions and structural results for the inventory-only problem, and in the second part we added price as a decision variable such that we analyzed a combined inventory & pricing problem. In both parts we considered the situation without and with positive shortage penalty cost separately, as the latter case causes additional technical difficulties.

In the first part of the work, where price is assumed not to affect the demand distribution with zero shortage penalty cost, we were able to prove the concavity of the optimization problem and we could derive simple, closed-form expressions for the optimal order quantity based on a transformation of the demand distribution according to the risk preferences. We were able to show that both the optimal cycle service level and the order quantity increase in the risk preference, meaning that they decrease as the decision maker becomes more risk-averse. This behaviour can be explained by saying that increasing order quantity increases the chance of higher profit realizations, but comes with a higher risk of more leftover inventory. This classical trade-off of the newsvendor model results in different optimal policies since, as risk preferences increase, the decision maker values the chance of higher realizations more than the risk of leftover inventory. An analysis of performance indicators such as expected profit, cycle service level and fill rate as two common service levels and the probability of loss as an internal indicator concluded the inventory problem with zero shortage penalty costs.

The inventory problem with positive shortage penalty costs is technically more demanding. The reason for this is that now random demand and profit realizations are no longer ordered in the same way, since the same low
profit realization can be caused either by high leftovers with low demand, or, alternatively, by high shortages with high demand. Because of this the risk measure of profit cannot be written directly in terms of the demand distribution. To overcome this problem either an optimization approach can be used, as proposed by Rockafellar and Uryasev (2002), or in our very specific situation we can take advantage of additional knowledge about profit realizations. As leftovers and shortages are two mutually exclusive events, we were able to sum up their probabilities after some rescaling and rewrite the problem as if there was no shortage penalty cost. Then we were again able to write the problem in terms of the demand distribution and show concavity results for the optimization. Although, in general, an explicit formulation for the optimal order quantity can no longer be found, the problem is reduced to a single-dimensional concave optimization problem for general risk spectra, while the general formulation by Acerbi (2002) results in an optimization problem with an infinite number of degrees of freedom. In the specific case of a piecewise constant risk spectrum we were able to formulate the problem as a system of non-linear equations which can be solved efficiently.

Using a numerical study we were able to conclude that the optimal order quantity is no longer monotone in the risk preference when positive shortage penalty costs must be considered. The explanation for this is that for high risk preferences the newsvendor will order more because he wants to increase the chance for higher profit realizations. On the other hand, as the newsvendor becomes very risk-averse, he is mainly concerned with shortage penalties, as these are generally not bounded from above. As a consequence, he will again increase his order quantity to hedge against the rare high demand events which cause extreme losses.

The second main part of the work was concerned with the combined inventory & pricing problem. Mean demand now depends on price. As is common in the related literature, we used the additive and the multiplicative demand models to combine the deterministic demand with the stochastic error. To ensure unimodality of the joint optimization problem we needed to restrict the risk spectra used to a subset which perserve certain properties. We can show by an example that the mean-CVaR$_\alpha$ risk spectrum does not preserve these properties and may result in multiple local price optima. Instead, the power risk spectrum satisfies all required conditions and turns out to be a flexible model as it can cover risk-averse, risk-neutral and risk-seeking preferences, and is implicitly a mean-deviation formulation already.

A main structural result for the combined problem concerned the price in the risk preference. For the additive model we were able to show that optimal price is increasing in the risk preference, while for the multiplicative
model numerical analysis shows that the optimal price is decreasing in the risk preference, under the condition that demand error variability is large enough. This result is particularly interesting as it shows the different strategies used by the newsvendor to hedge against or deal with demand uncertainty, since in the additive model the coefficient of variation of demand increases in price (with constant variance), while the variance decreases in price for the multiplicative model (with constant coefficient of variation).

Considering also shortage penalty costs for the joint problem resulted clearly in the technically most challenging model. In contrast to the previous models, here we were no longer able to derive results analytically; instead, we conducted a numerical study in order to gain insights into this problem. For unimodality of the problem we can numerically identify the same conditions on the distribution functions and risk spectra as for the problem with zero shortage penalty costs. The optimum price in the risk preference is again increasing for the additive and decreasing for the multiplicative demand model, while optimal quantities are non-monotone for both cases. Positive shortage penalty costs also affect the service levels because in the extremely risk-averse case, both measures approach one.

There are plenty of opportunities for further extension of the work. One extension could be with respect to the estimation of the underlying demand model. Typically, linear or log-linear regression is used to estimate the response of demand on price. The commonly used least-squares minimization treats all observations equally so that the regression model might be good on average but for the rather rare outcomes with bad consequences the regression model might not explain demand very well. Hence, similar to applying spectral risk measures, it could be interesting to apply weighted regression for the demand modeling to be able to specifically estimate the lower tails of the demand distribution.

While the current work is strongly based on a normative foundation, a positive study about how good different risk spectra might reflect empirically observable decision making behaviour could be very helpful. Based on these results the model could be used for supply chain contracting issues, for example where a single manufacturer delivers to multiple risk-averse retailers, where the manufacturer needs to anticipate the response of the retailers on the pricing decision. Furthermore, additional research on technical properties of risk spectra could be done in order to find risk spectra other than CVaR and power risk spectrum, where the necessary conditions for the pricing problem are fulfilled.

A challenging task could be the extension of the model to a multi-product setting. Choi and Ruszczyński (2008) and Choi et al. (2009) analyze approx-
imation techniques for quantity optimization when a product portfolio is considered under general law-invariant coherent risk measures. A main conclusion of their work is that the whole product portfolio has to be considered in the optimization when risk measures are applied. To our knowledge, work with respect to the pricing problem in such a setting has not yet been done.

A natural further extension of this work concerns dynamic multi-period models. It would be very interesting to see if, for the inventory-only problem, a basestock policy, or in the inventory & pricing problem, a basestock listprice policy, turns out to be optimal.