Appendix A

Proofs

Lemma 8. (See Jammernegg and Kischka, 2007, appendix B) Let $F$ be the continuous, strictly increasing distribution function of demand $D$. The distribution function of profit, $F_{\Pi}$ is

$$F_{\Pi}(\pi) = \begin{cases} F \left( \frac{\pi + y(c-v)}{p-v} \right) & \text{for } \pi < (p-c)y \\ 1 & \text{otherwise}, \end{cases} \quad (A.1)$$

so that $F_{\Pi}$ is continuous and strictly increasing for $\pi < (p-c)y$. The generalized inverse distribution function of profit, $F_{\Pi}^{-1}(\omega)$, is then strictly increasing for $\omega \in [0,F(y))$ and $(p-c)y$ for $\omega \in [F(y), 1]$.

Proof. Using the profit formulation as in (1.1), $\Pi(y) = p \min(D, y) - cy + v(y-D)^+$, one can easily see that with a given order quantity $y$ when $D = y$, a maximum possible profit of $(p-c)y$ can be achieved.

Case 1: $\pi > (p-c)y$. For any $D > y$, no further profit improvements can be made. Hence,

$$F_{\Pi}(\pi) = 1 \quad \text{for } D > y.$$

Case 2: $\pi \leq (p-c)y$. For the case $D \leq y$, random profit can be written as

$$\Pi = pD - cy + (y-D)v.$$

Exchanging variables,

$$F_{\Pi}(\pi) = \mathbb{P}(\Pi \leq \pi)$$

$$= \mathbb{P}(pD - cy + (y-D)v \leq \pi)$$

$$= \mathbb{P} \left( D \leq \frac{\pi + y(c-v)}{p-v} \right)$$

$$= F \left( \frac{\pi + y(c-v)}{p-v} \right).$$
Proof of Proposition 4

Since for the problem without penalty cost an ordered relation between demand realizations and profit realizations exists (i.e. the $100\alpha\%$ lowest demand result in the $100\alpha\%$ lowest profit for any $\alpha$), we can use the definition of the risk measure as in (2.9) directly for the optimization. Hence, we can write the objective function, $M(\Pi(y))$, as

$$M(\Pi(y)) = \int_0^{F(y)} \phi(\omega) F_{\Pi}^{-1}(\omega) d\omega + (p - c)y \int_{F(y)}^1 \phi(\omega) d\omega.$$

Let $\Phi(\omega) := \int_0^\omega \phi(u) du$, then

$$M(\Pi(y)) = \int_0^{F(y)} \phi(\omega) F_{\Pi}^{-1}(\omega) d\omega + (p - c)y (1 - \Phi(F(y))),$$

where $F_{\Pi}^{-1}(\omega)$ for $\omega \in [0, F(y))$ using Lemma 8 is a continuous, monotone increasing function. Hence, we can change the direction of integration and write

$$M(\Pi(y)) = \int_{-(p-c)y}^{(p-c)y} \pi \phi(F_{\Pi}(\pi)) dF_{\Pi}(\pi) + (p - c)y (1 - \Phi(F(y))).$$

Replacing profit distribution $F_{\Pi}$ with demand distribution $F$ using Lemma 8 leads to the formulation of the risk measure for the newsvendor problem,

$$M(\Pi(y)) = \int_0^y [xp - cy + (y - x)v] \phi(F(x)) dF(x) + (p - c)y [1 - \Phi(F(y))].$$

We are now ready to derive the first order condition to explicitly formulate the optimal order quantity, $y^*$. Using Leibnitz’ rule,

$$\frac{dM}{dy} = -(c + v) \int_0^y \phi(F(x)) dF(x) + (p - c)[1 - \Phi(F(y))] = 0,$$

Solving for $\Phi(F(y))$,

$$\Phi(F(y)) = \frac{p - c}{p - v}.$$

Note that $\Phi^{-1}(\omega)$ exists for every $\omega \in [0, 1]$ since $\phi(\omega)$ is finite by definition.
Finally, we derive the second order condition to show that the optimization problem is concave in $y$, so
\[
\frac{d^2 M}{dy^2} = -(c - v)\phi(F(y))f(y) - (p - c)\phi(F(y))f(y) < 0,
\]
since $v < c < p$ and $\phi(\cdot) \geq 0$, $f(\cdot) \geq 0$ by definition. Hence, the problem is a concave maximization problem in $y$. \qed

**Proof of Lemma 2**

Using (1.9) and (1.1) the probability of profit being smaller than a target level $L$ is
\[
PL_L = \mathbb{P}(\Pi \leq L) = \mathbb{P}((p - c)y - (p - v)(y - D)^+ \leq L)
\]
\[
= \mathbb{P}\left(\max(y - D; 0) \geq \frac{(p - c)y - L}{p - v}\right)
\]
\[
= \begin{cases} 
\mathbb{P}\left(D \leq \frac{(c - v)y + L}{p - v}\right) & \text{for } (p - c)y - L \geq 0 \\
1 & \text{for } (p - c)y - L < 0 
\end{cases}
\]
\[
= \begin{cases} 
F\left(\frac{(c - v)y + L}{p - v}\right) & \text{for } (p - c)y - L \geq 0 \\
1 & \text{for } (p - c)y - L < 0. 
\end{cases}
\]

It can be easily seen that for any $y \geq \frac{L}{p - c}$, $PL_L$ is monotonically increasing in $y$ since $F^{-1}$ is increasing in its argument. \qed

**Proof of Lemma 3**

The probability of profit being smaller a certain level, $\mathbb{P}(\Pi \leq \pi)$ is composed of two parts: the event where $D \leq y$ and a second event where $D > y$. Note that these two events are mutually exclusive, therefore for the joint probability we can simply add up the probabilities of the two events. Recall from (3.17),
\[
\Pi(y) = \begin{cases} 
(p - c)D - (c - v)(y - D) & D \leq y \\
(p - c)y - s(D - y) & D > y. 
\end{cases}
\]

Now we can plug in the profit in the distribution function of profit, $F_{\Pi}(\pi) := \mathbb{P}(\Pi \leq \pi)$, and add up the probabilities of the two exclusive demand events,
so that we can express the profit distribution $F_{II}$ as a function of the demand distribution $F$.

$$F_{II}(\pi) = \mathbb{P} \left( \Pi(y) \leq \pi, D \leq y \right) + \mathbb{P} \left( \Pi(y) \leq \pi, D > y \right)$$

$$= \mathbb{P} \left( (p - c)D - (c - v)(y - D) \leq \pi, D \leq y \right) + \mathbb{P} \left( (p - c)D - (c - v)(y - D) \leq \pi, D > y \right)$$

$$= \mathbb{P} \left( D \leq \frac{\pi + (c - v)y}{p - v} \right) + \mathbb{P} \left( D > \frac{(p - c + s)y - \pi}{s} \right)$$

$$= F \left( \frac{\pi + (c - v)y}{p - v} \right) + 1 - F \left( \frac{(p - c + s)y - \pi}{s} \right).$$

\[ \square \]

**Proof of Proposition 8**

Using Acerbi’s method easily helps us to show the concavity of $M(\Pi(y))$ with respect to $y$. We can formulate the risk measure as in Proposition 3, so

$$M(\Pi(y)) = \max_{\psi} \Gamma(y, \psi),$$

where $\Gamma$ is defined in (2.14). Since random profit $\Pi(y)$ is concave in $y$, immediately from Corollary 2 it follows that $M(\Pi(y))$ is concave in the order quantity.

It remains to derive the risk measure in terms of demand. Since $(p - c)y$ is the maximum possible profit realization for a given $y$, we can write the risk measure in terms of the profit distribution as

$$M(\Pi(y)) = \int_{-\infty}^{(p-c)y} t d\Phi(F_{II}(t)).$$

Exchanging the variable of integration $t$ over profits by demand $x$, it follows that

$$M(\Pi(y)) = \int_{-\infty}^{y} \pi(x, y) d\Phi(G(x))$$

$$= \int_{-\infty}^{y} ((p - v)x - (c - v)y) \left( f(x) + f(\bar{x}) \frac{p - v}{s} \right) \phi(F(x) + 1 - F(\bar{x})) dx.$$
Using integration by parts,

\[ M(\Pi(y)) = (p - c)y - (p - v) \int_{-\infty}^{y} \Phi(F(x) + 1 - F(\bar{x})) \, dx. \]

\[
\]

\[
\]

Proof of Lemma 4

Recall that for a CVaR\(\_\alpha\) decision maker \(\phi = \frac{1}{\alpha}\) for \(0 \leq \omega \leq \alpha\) and 0 otherwise. Now, let \(x^o < y\) be the demand up to which the corresponding profits are considered by having positive weights, so where \(\phi(F_{\Pi}) = \frac{1}{\alpha}\). The corresponding demand level larger \(y\), i.e. \(\bar{x}^o\), can be derived as shown in (3.18), so

\[ \bar{x}^o = y + (y - x^o)\frac{p - v}{s}. \]

Hence, \(x^o\) should satisfy

\[ G(x^o) = F(x^o) + 1 - F(\bar{x}^o) = \alpha. \tag{A.2} \]

Since \(\bar{x}^o\) depends on \(y\), when \(y\) changes also \(x^o\) should change so that (A.2) is satisfied again. Thus \(x^o\) is implicitly a function of \(y\). Now we can define \(x'^{ot} = \frac{dx^o}{dy}\) and \(\bar{x}'^{ot} = \frac{d\bar{x}^o}{dy}\), taking the derivative of (A.2) with respect to \(y\) leads to

\[ x'^{ot}f(x^o) - \bar{x}'^{ot}f(\bar{x}^o) = 0. \tag{A.3} \]

Based on the general formulation of \(M(\Pi)\) in (3.23) we can write

\[ M(\Pi(y)) = \frac{1}{\alpha} \int_{-\infty}^{x^o} ((p - c)x - (c - v)(y - x))f(x)\frac{p - v}{s} \, dx \\
+ \frac{1}{\alpha} \int_{-\infty}^{\bar{x}^o} ((p - c)x - (c - v)(y - x))f(x) \, dx. \]

Substituting \(\bar{x}\) for \(x\) in the first integral leads to

\[ M(\Pi(y)) = \frac{1}{\alpha} \int_{\bar{x}^o}^{\infty} ((p - c)y - s(\bar{x} - y))f(\bar{x}) \, d\bar{x} \\
+ \frac{1}{\alpha} \int_{-\infty}^{x^o} ((p - c)x - (c - v)(y - x))f(x) \, dx, \]
taking the derivative with respect to $y$ using (A.3),

$$M'(\Pi(y)) = \frac{1}{\alpha} \left( (p-c+s)(1-F(\bar{x}^o)) - (c-v)F(x^o) \right).$$

Using (A.2) we can solve this for

$$F(x^{o*}) = \alpha \frac{p-c+s}{p-v+s}, \quad F(\bar{x}^{o*}) = 1 - \alpha \frac{c-v}{p-v+s},$$

and

$$y^* = \frac{p-v}{p-v+s} F^{-1} \left( \alpha \frac{p-c+s}{p-v+s} \right) + \frac{s}{p-v+s} F^{-1} \left( 1 - \alpha \frac{c-v}{p-v+s} \right).$$

\[\square\]

**Proof of Lemma 6**

Following the same line of argument as in the proof of Corollary 4, we now define $J$ demand levels as

$$\bar{x}_i^o = y + (y - x_i^o) \frac{p-v}{s} \quad \text{for all } i = 1 \ldots J,$$

and $x_0^o := -\infty$ and $x_{J+1}^o := y$, so $\bar{x}_0^o = \infty$ and $x_{J+1}^o = y$ which satisfy

$$F(x_i^o) + 1 - F(\bar{x}_i^o) = \omega_i \quad \text{for all } i = 1 \ldots J.$$

The risk measure can be formulated as

$$M(\Pi(y)) = \sum_{i=1}^{J+1} \phi_i \left[ \int_{\bar{x}_i^o}^{x_i^o} ((p-c)y - s(\bar{x} - y)) f(\bar{x}) d\bar{x} \\
+ \int_{x_i^o}^{x_{i+1}^o} ((p-c)x - (c-v)(y-x)) f(x) dx \right].$$
Proofs

Taking derivatives leads to the first order condition as the system of equations

\[
\frac{dM(\Pi(y))}{dy} = \sum_{i=1}^{J+1} \phi_i \left[ (p - c + s) \left( F(\bar{x}_{i-1}^o) - F(\bar{x}_i^o) \right) 
- (c - v) \left( F(x_i^o) - F(x_{i-1}^o) \right) \right] = 0,
\]

\[
F(x_i^o) + 1 - F(\bar{x}_i^o) = \omega_i \quad \text{for all } i. \]

\[\square\]

Proof of Proposition 11

For the additive model, we write the objective function as

\[
M(\Pi(p, z^*(p))) = M(\Pi(p)) = (p-c)d(p) + (p-c)z^*(p) - (p-v) \int_{-\infty}^{z^*(p)} F_\phi(\varepsilon) \, d\varepsilon,
\]

with cross derivative

\[
\frac{\partial^2 M(\Pi(p))}{\partial v \partial p} = \frac{dz^*(p)}{dv} \left( 1 - \frac{p-c}{p-v} \right) > 0.
\]

The second term is positive by definition since \( p \geq c \geq v \). To see that \( z^*(p) \) is increasing in \( v \), we take the derivative with respect to \( v \) from \( F_\phi(z^*(p)) = \frac{p-c}{p-v} \),

\[
\frac{dz^*(p)}{dv} = \frac{1}{f_\phi(z^*(p))} \frac{p-c}{(p-v)^2} > 0.
\]

Hence, the risk measure of profit is supermodular in \( (p^*, v) \), so \( p^* \) is increasing in \( v \). \[\square\]

Proof of Proposition 12

We write the objective function in terms of the degree of risk aversion, \( \eta \) using the optimal stocking factor \( z^*(p) \). Note that the risk-transformed
distribution function $F_\phi(\epsilon)$ changes with $\eta$. Let us first show part (a) for the additive model.

$$M(\Pi(p, z^*(p))) = M(\Pi(p)) = (p - c)(d(p) + z^*(p)) - (p - v) \int_{-\infty}^{z^*(p)} F_\phi(\epsilon) \, d\epsilon.$$  

The cross derivative is

$$\frac{\partial^2 M(\Pi(p))}{\partial \eta \partial p} = \frac{\partial z^*(p)}{\partial \eta} (1 - F_\phi(z^*(p))) - \int_{-\infty}^{z^*(p)} \frac{\partial F_\phi(\epsilon)}{\partial \eta} \, d\epsilon, \quad (A.4)$$

where $F_\phi(z^*(p)) = \frac{p - c}{p - v}$. By Definition 6(b), $\Phi(\omega)$ decreases in $\eta$, hence also $F_\phi(\epsilon) = \Phi(F(\epsilon))$ decreases in $\eta$. Furthermore, the optimal $z^*(p) = F^{-1}_\phi \left( \frac{p - c}{p - v} \right)$ for a given $p$ increases in $\eta$. Hence, since $0 \leq F_\phi(z^*(p)) \leq 1$ the first term in (A.4) is positive, while the second term is negative so that the whole expression is positive. This is sufficient for $M(\Pi(p))$ being supermodular in $(p^*, \eta)$ and $p^*$ being increasing in $\eta$.

Now we can show part (b) for the multiplicative model. The first derivative with respect to price is

$$\left. \frac{\partial M(\Pi(p))}{\partial p} \right|_{p=p^*} = d'(p^*) \left[ p^* \int_0^{z^*(p^*)} (1 - F_\phi(\epsilon)) \, d\epsilon - cz^*(p^*) \right]$$

$$+ d(p^*) \int_0^{z^*(p^*)} (1 - F_\phi(\epsilon)) \, d\epsilon = 0,$$

hence

$$d'(p^*) = - \frac{d(p^*) \int_0^{z^*(p^*)} (1 - F_\phi(\epsilon)) \, d\epsilon}{p^* \int_0^{z^*(p^*)} (1 - F_\phi(\epsilon)) \, d\epsilon - cz^*(p^*)}.$$

Recall that $\epsilon(p)$ denotes the price elasticity,

$$\epsilon(p^*) = - \frac{p^* d'(p^*)}{d(p^*)} = p^* \frac{\int_0^{z^*(p^*)} (1 - F_\phi(\epsilon)) \, d\epsilon}{p^* \int_0^{z^*(p^*)} (1 - F_\phi(\epsilon)) \, d\epsilon - cz^*(p^*)}, \quad \text{and}$$

$$1 - \epsilon(p^*) = \frac{d(p^*) - p^* d'(p^*)}{d(p^*)} = \frac{-cz^*(p^*)}{p^* \int_0^{z^*(p^*)} (1 - F_\phi(\epsilon)) \, d\epsilon - cz^*(p^*)}. \quad (A.5)$$
Using integration by parts, we can rewrite the denominator as

\[ 1 - \epsilon(p^*) = \frac{-cz^*(p^*)}{p^* \int_0^{z^*(p^*)} \epsilon f_\phi(\epsilon) \, d\epsilon}. \]  
(A.6)

The cross derivative is

\[
\frac{\partial^2 M(\Pi(p))}{\partial \eta \partial p} = \\
d'(p) \left[ -\int_0^{z^*(p)} \frac{\partial F_\phi(\epsilon)}{\partial \eta} \, d\epsilon + p \frac{\partial z^*(p)}{\partial \eta} (1 - F_\phi(z^*(p))) - cz^*(p) \right]
+ \frac{d(p)}{d(p/z^*(p))} \left[ \int_0^{z^*(p)} \frac{\partial F_\phi(\epsilon)}{\partial \eta} \, d\epsilon + \frac{\partial z^*(p)}{\partial \eta} (1 - F_\phi(z^*(p))) \right],
\]

where \( F_\phi(z^*(p)) = \frac{p-c}{p} \), so

\[
\frac{\partial^2 M(\Pi(p))}{\partial \eta \partial p} = -\int_0^{z^*(p)} \frac{\partial F_\phi(\epsilon)}{\partial \eta} \, d\epsilon \cdot \left( d(p) + pd'(p) \right) + d(p) \frac{\partial z^*(p)}{\partial \eta} \frac{c}{p}
\]

Since \( d(p^*) + p^*d'(p^*) = d(p^*)(1 - \epsilon(p^*)) \) from (A.5), we can write

\[
\left. \frac{\partial^2 M(\Pi(p))}{\partial \eta \partial p} \right|_{p=p^*} = \\
d(p^*) \left[ -(1 - \epsilon(p^*)) \int_0^{z^*(p^*)} \frac{\partial F_\phi(\epsilon)}{\partial \eta} \, d\epsilon + \frac{\partial z^*(p)}{\partial \eta} \right]_{p=p^*} \cdot \frac{c}{p^*}.
\]
Plugging in (A.6),

$$\left. \frac{\partial^2 M(\Pi(p))}{\partial \eta \partial p} \right|_{p=p^*} = d(p^*) \left[ \frac{c}{p^*} \int_0^{z^*(p^*)} \frac{\partial F_\phi(\varepsilon)}{\partial \eta} d\varepsilon \right] + \left. \frac{\partial z^*(p)}{\partial \eta} \right|_{p^*} \cdot \left[ \frac{c}{p^*} \int_0^{z^*(p^*)} \varepsilon f_\phi(\varepsilon) d\varepsilon \right].$$

Since

$$\frac{\partial z^*(p)}{\partial \eta} = - \left. \frac{\partial F_\phi(\varepsilon)}{\partial \eta} \right|_{\varepsilon=z^*(p)} \times \frac{1}{f_\phi(z^*(p))},$$

we can write

$$\left. \frac{\partial^2 M(\Pi(p))}{\partial \eta \partial p} \right|_{p=p^*} = d(p^*) \frac{c}{p^*} \left( \int_0^{z^*(p^*)} \varepsilon f_\phi(\varepsilon) d\varepsilon \right)^{-1} \frac{1}{f_\phi(z^*(p^*))} \times \left[ z^*(p^*) f_\phi(z^*(p^*)) \int_0^{z^*(p^*)} \frac{\partial F_\phi(\varepsilon)}{\partial \eta} d\varepsilon - \frac{\partial G(z^*(p^*))}{\partial \eta} \int_0^{z^*(p^*)} \varepsilon f_\phi(\varepsilon) d\varepsilon \right],$$

where the first terms are all positive and the expression in $[\cdot]$ can be written as

$$\int_0^{z^*(p^*)} \frac{\partial F_\phi(\varepsilon)}{\partial \eta} \frac{\partial F_\phi(z^*(p^*))}{\partial \eta} \left\{ \frac{z^*(p^*) f_\phi(z^*(p^*))}{\partial F_\phi(z^*(p^*))} - \varepsilon f_\phi(\varepsilon) \right\} d\varepsilon. \quad (A.7)$$

This expression is positive if

$$\frac{d}{dx} \left( \frac{xf_\phi(x)}{\partial \eta F_\phi(x)} \right) > 0,$$

so if the second fraction in (A.7) is increasing in $\varepsilon$. \qed
Proof of Proposition 13

Let us first show the monotonicity of \( p^*(y) \) for an additive demand model. Note that for ease of expression we show the derivations without considering salvage value, so \( v = 0 \). It can be easily extended to \( v > 0 \). We can write

\[
M(\Pi(p, y)) = \int_{-d(p)}^{y-d(p)} p(d(p) + \varepsilon)f_{\phi}(\varepsilon) \, d\varepsilon + \int_{y-d(p)}^{\infty} p(y)f_{\phi}(\varepsilon) \, d\varepsilon - cy.
\]

Using \( y = d(p) + z \),

\[
M(\Pi(p, y)) = (p - c)y - p \int_{-d(p)}^{y-d(p)} (z - \varepsilon)f_{\phi}(\varepsilon) \, d\varepsilon.
\]

Using integration by parts,

\[
M(\Pi(p, y)) = (p - c)y - p \int_{-d(p)}^{y-d(p)} F_{\phi}(\varepsilon) \, d\varepsilon.
\]

The first order condition for optimal \( p^*(y) \) is then

\[
\frac{\partial M(\Pi(p, y))}{\partial p} \bigg|_{p=p^*(y)} = \int_{-d(p)}^{y-d(p)} [1 - F_{\phi}(\varepsilon) + pd'(p)f_{\phi}(\varepsilon)] \, d\varepsilon \bigg|_{p=p^*(y)} = 0. \tag{A.8}
\]

Note that \( F_{\phi}(-d(p)) = 0 \). Using the implicit function theorem,

\[
\frac{dp^*(y)}{dy} = -\frac{\partial^2 M(\Pi(p, y))}{\partial y \partial p} \left( \frac{\partial^2 M(\Pi(p, y))}{\partial p^2} \right)^{-1} \bigg|_{p=p^*(y)},
\]

where the second term is negative because of the second order condition for optimality of \( p^*(y) \). For a decreasing \( p^*(y) \) it remains to show that the first term is negative. It can be written as

\[
\frac{\partial^2 M(\Pi(p, y))}{\partial y \partial p} = 1 - F_{\phi}(y - d(p)) + pd'(p)f_{\phi}(y - d(p)) \tag{A.9}
\]

Now, let \( R(p, \varepsilon) := -pd'(p) \frac{f_{\phi}(\varepsilon)}{1 - F_{\phi}(\varepsilon)} \) so that

\[
1 - F_{\phi}(\varepsilon) + pd'(p)f_{\phi}(\varepsilon) = (1 - F_{\phi}(\varepsilon))(1 - R(p, \varepsilon)).
\]

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Appendix A. Proofs

Now we show that for \( p = p^*(y) \), (A.9) is negative by contradiction. Assume

\[
(1 - F_\phi(\varepsilon))(1 - R(p, \varepsilon))\big|_{\varepsilon = y - d(p)} \geq 0.
\]

Because \((1 - F_\phi(\cdot)) \geq 0\) it follows that \(1 - R(p, \varepsilon)\big|_{\varepsilon = y - d(p)} \geq 0\). \(R(p, \varepsilon)\) is increasing in \( \varepsilon \), since \( F \) has IFR and the risk spectrum preserves the IFR property, which implies that \( \frac{f_\phi(\varepsilon)}{1 - F_\phi(\varepsilon)} \) is increasing in \( \varepsilon \). Hence, \(1 - R(p, \varepsilon) > 0\) for any \( \varepsilon \in (-d(p), y - d(p)) \). It follows that

\[
\int_{-d(p)}^{y-d(p)} (1 - F_\phi(p, \varepsilon))(1 - R(p, \varepsilon)) \, d\varepsilon \bigg|_{p=p^*(y)} > 0,
\]

which is a contradiction to (A.8).

The proof for the multiplicative demand function is very similar. The cross derivative is

\[
\frac{\partial^2 M(\Pi(p, y))}{\partial y \partial p} = 1 - F_\phi \left( \frac{y}{d(p)} \right) + \frac{pd'(p)}{d(p)} \frac{y}{d(p)} f_\phi \left( \frac{y}{d(p)} \right). \tag{A.10}
\]

We can define \( R(p, \varepsilon) := \frac{pd'(p)}{d(p)} \frac{\varepsilon f_\phi(\varepsilon)}{1 - F_\phi(\varepsilon)} \) which is increasing in \( \varepsilon \). Using the same argument by contradiction as before, \( (1 - F_\phi(\varepsilon))(1 - R(p, \varepsilon))\big|_{\varepsilon = \frac{y}{d(p)}} < 0 \), which implies that (A.10) is negative for \( p = p^*(y) \) and \( p^*(y) \) is decreasing in \( y \).

Proof of Corollary 6

It is easy to see that the risk measure of the profit from the demand error,

\[
M(\Pi_\varepsilon(p, z)) = (p - c)z - (p - v) \int_{-\infty}^{z} F_\phi(\varepsilon) \, d\varepsilon,
\]

is supermodular in \((p, z)\), since

\[
\frac{\partial^2 M(\Pi_\varepsilon(p, z))}{\partial z \partial p} = 1 - F_\phi(z) > 0.
\]

Hence, independent of the underlying demand model \( p^* \) is increasing in \( z \). \( \square \)
Proof of Corollary 7

Let us first show the behaviour for the additive demand model. We write the risk measure of profit as a function of \( p \) and \( \eta \),

\[
\text{M}(\Pi(p, z)) = (p - c)(d(p) + z) - (p - v) \int_{-\infty}^{z} F_\phi(\varepsilon) \, d\varepsilon,
\]

with cross derivative,

\[
\frac{\partial^2 \text{M}(\Pi(p, z))}{\partial p \partial \eta} = - \int_{-\infty}^{z} \frac{\partial}{\partial \eta} F_\phi(\varepsilon) \, d\varepsilon > 0.
\]

This holds since by Definition 6(b) \( F_\phi \) is decreasing in \( \eta \). Hence, the risk measure is supermodular in \((p^*, \eta)\) for a given \( z \) and \( p^*(z) \) is increasing in \( \eta \).

Now we can show the behavior for the multiplicative demand model. Here the risk measure of profit is

\[
\text{M}(\Pi(p, z)) = d(p) \left[(p - c)z - (p - v) \int_{0}^{z} F_\phi(\varepsilon) \, d\varepsilon\right],
\]

with cross derivative

\[
\frac{\partial^2 \text{M}(\Pi(p, z))}{\partial p \partial \eta} = - \left(d'(p)(p - v) + d(p)\right) \int_{0}^{z} \frac{\partial}{\partial \eta} F_\phi(\varepsilon) \, d\varepsilon. \quad (A.11)
\]

The integral is with negative the same reasoning as before in the additive model. What remains, is to show that the first term is negative. The first order condition for optimality of price is

\[
\frac{\partial \text{M}(\Pi(p, z))}{\partial p} = - \left(d'(p)(p - v) + d(p)\right) \int_{0}^{z} F_\phi(\varepsilon) \, d\varepsilon + z \left(d'(p)(p - c) + d(p)\right) \bigg|_{p=p^*(z)} = 0.
\]

Since \( p - c < p - v \), we can write

\[
\left(d'(p)(p - v) + d(p)\right) \left(z - \int_{0}^{z} F_\phi(\varepsilon) \, d\varepsilon\right) < 0.
\]
Since \( F_{\phi}(\cdot) \leq 1 \), the integral is smaller than \( z \) and the right term is positive. Hence, the first term is negative, so that also (A.11) is negative. The problem is submodular in \((p^*, \eta)\), so \( p^*(z) \) is decreasing in \( \eta \).
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Band 11 Birgit Trukeschitz: Im Dienst Sozialer Dienste. Ökonomische Analyse der Beschäftigung in sozialen Dienstleistungseinrichtungen des Nonprofit Sektors. 2006


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<td>Rainer Quante: Management of Stochastic Demand in Make-to-Stock Manufacturing.</td>
<td>2009.</td>
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Gerhard Wohlgenannt: Learning Ontology Relations by Combining Corpus-Based Techniques and Reasoning on Data from Semantic Web Sources. 2011.

Thomas Peschta: Der Einfluss von Kundenzufriedenheit auf die Kundenloyalität und die Wirkung der Wettbewerbsintensität am Beispiel der Gemeinschaftsverpflegungsgastronomie. 2011.


www.peterlang.de
The price-setting newsvendor model is used to address the single period joint pricing and inventory control problem. The objective is to set the optimal price and replenishment quantity of a single product in order to maximize the expected profit. Products with a short selling season and relatively long replenishment lead times such as fashion goods are the most relevant application areas of the model. The focus of the work is the generalization of the model with respect to the modeling of uncertainty in demand. The author presents an analytical and empirical study which compares different demand models with a more flexible model based on price and inventory optimization. She concludes that using a general model can increase the profits significantly.

Content: Inventory management · Review of the newsvendor model · Price-taking and price-setting newsvendor model · Empirical study · Simulation of profits · Analysis of the generalized model · Elasticity of expected sales · Optimality conditions · Structural properties · Numerical study