IV Analytical Agency Models

1 Overview

In the Section (2.1), a basic model is set up to analyse the situation wherein effort is uncontractible. It will be seen that parties will switch to output contracting. In the third Section (2.2) this very general result will be explicitly modelled by making rather strong assumptions about distributions, utility functions and the structure of the incentive scheme. Subsequently, in Section (2.3) a closer look is taken at risk-sharing, still within this very explicit framework. Section (2.4) will be general again. The intention is to expose the mechanics of the optimal sharing rule. This helps one to understand why relatively strong assumptions are needed to derive meaningful results. Robustness becomes an issue. Section (2.5) discusses some limitations of the presented models. Dealing with these limitations is the objective of subsequent Chapters.

Chapter (3) deals with the consequences of error in judgement and bankruptcy in both input monitoring and output monitoring models. Section (3.1) shows that input monitoring can theoretically achieve first best if harsh enough punishment is feasible and error in judgement can be excluded. Yet, if a bankruptcy constraint is introduced input monitoring will be costly. Alternatively, if error in judgement is allowed for, input monitoring will be costly even if there is no bankruptcy constraint. Section (3.2) deals with shifting support schemes which allow perfectly accurate output monitoring and achieve first best beyond the obvious case of a deterministic production function. Another problem considered will be the moral hazard with respect to risk which might arise in output monitoring schemes in the presence of bankruptcy constraints. In both cases the argument is only sketched.

Chapter (4) deals with transaction cost and distortion. Section (4.1) describes sources of direct and indirect transaction cost. Section (4.2) discusses the problem of distortion which arises if there is a tension between what the principal wants and what the agent is rewarded for. It can be shown that distortion can be divided into two components: scaling and alignment. There is conflict with the risk-incentive trade-off as output monitoring is less distortive but generally more prone to error.

Many traditional models of contract theory are one period. The subject of Chapter (5) will be to analyse the effect of time on contracts. The starting point of this discussion is the often stated thesis that time can resolve incentive issues that arise in one-shot relationships costlessly. Four models are presented: The first model deals with the advantage of long-term contracts over short term contracts.
Time allows lowering the cost of incentives by reducing imperfect risk-sharing of output-based contracts (5.2). The second and third model deal with situations wherein relational contracts solve problems of enforcement. The theory of supergames will be used to argue that time may sustain contracts with otherwise desirable properties, which would not be feasible in a one-shot relationship. This is the case wherein contract parameters are observable but not verifiable (5.3). The forth model introduces career concerns which induce the agent to exert effort although choice of effort cannot be contracted on. It will be shown that an implicit contract links the agent’s current choice of effort to future pay-off. (5.4). In conclusion, it will be argued that the thesis that time solves incentive issues costlessly cannot be generally upheld. Time merely alters and enriches the insights from one-period models: Conclusions from the one-period models are not necessarily valid in the multi-period settings.

2 The Classical Risk-Incentive Trade-Off

2.1 The Basic Model

2.1.1 Introduction

A model will be set up to study optimal pay incentives in the principal-agent relationship. The following propositions will be derived:

1. If effort is contractible, compensation should not be made contingent on output if it is assumed that the agent is risk-averse and the principal risk-neutral. More generally, if effort is contractible, there is no rationale for output-based schemes in order to avoid shirking. The risk-sharing argument comes fully to bear.

2. If there is a stochastic relationship between effort and output and effort is not contractible compensation should be made contingent on output. Incentives rise in strength as compensation differentials increase.

2.1. First best can be achieved if the agent is risk neutral. He becomes residual claimant.

2.2. In the case of stochastic production functions and a risk-averse agent, only a second best solution can be achieved due to the risk-incentive trade-off.

3. In the case of a deterministic production function, output-based schemes achieve first best. A forcing contract can be used.
2.1.2 Modeling Assumptions

It is assumed that there are two output levels, low output $x_l$ and high output $x_h$. Effort can be chosen on a continuous interval within two bounds, $e \in [e_l, e_h]$. Output stochastically depends on effort. The uncertainty is captured by a probability distribution over output levels, $P(x = x_l) = 1 - p$, $P(x = x_h) = p$. Effort $e$ is a parameter of the probability distribution, $p = p(e)$. As it is assumed that any outcome is possible under any action, the range of $p(e)$ is the open interval $(0, 1)$. It is further assumed that the probability of high output strictly increases with increased levels of effort, but less than proportionately. So, formally, $p(e)$ is a strictly increasing and concave function in $e$ ($p'(\cdot) > 0$, $p''(e) < 0$) on the open interval $(e_l, e_h)$. In order to avoid boundary solutions, $p'(e_l) = \infty$ and $p'(e_h) = 0$. Preferences over lotteries for both the principal and the agent obey the von-Neumann-Morgenstern (v-N-M) axioms.

The agent's Bernoulli utility function $u(w, e)$ is known to the principal and depends on the...
pay level \( w_i = w(x_i) \) where \( i = \{l, h\} \) and on his effort choice \( e \). It is further assumed that \( u(w, e) \) is additively separable\(^{268}\) in a part that depends on the pay level and another part which depends on his choice of effort and that the agent likes money and dislikes effort. Money utility for both the agent and the principal are given by functions \( u(\cdot) \) which are strictly increasing, continuously differentiable and concave, which means that they are either risk-averse or risk neutral. The agent’s disutility of effort \( d(e) \) is usually assumed to be strictly increasing and convex in \( e \). The principal offers a contract to the agent who will either accept or reject it, in which case the agent’s utility is his reservation utility \( u_0 \)\(^{269}\) and the principal’s utility is 0.

### 2.1.3 Contractible Effort

The crucial point in the subject of this thesis is to study the effect of asymmetric information on contracting. Contrary to this assumption it shall be assumed — as a benchmark case — that the effort decision of the agent is perfectly observable by both, the agent and the principal and therefore the contract can stipulate effort as a contingency.

If the parties can contract on effort, the principal, when designing his optimal compensation scheme, has to solve the following optimization problem:

\[
\max_{\gamma(e, w_l, w_h)} \Pi(\gamma) = p(e)(x_h - w_h) + (1 - p(e))(x_l - w_l) \tag{1.1}^{270}
\]

\[\text{s.t. } U(e, w_l, w_h) = p(e)u(w_h) + (1 - p(e))u(w_l) - e \geq 0 \tag{1.2}\]

The **principal is risk-neutral**. He is maximizing expected pay-off. The **agent is assumed to be risk-averse**. Condition (1.2) is called the agent’s participation constraint which will hold with equality for the solution because otherwise the principal could lower the compensation for the agent and still

\(^{268}\) This assumption was shown to be crucial by Gjesdal (1982)

\(^{269}\) Market forces ensure that the agent can earn his reservation utility elsewhere.

\(^{270}\) It is common to divide this problem into steps (see discussion at the end of the Section). The first step would be to find out the minimum cost incentive scheme to implement any given effort level \( e \). Mathematically, the second step would be to choose the effort level which maximizes net benefit for the principal. For the implication derived in this Section it is sufficient to solve the first step. Therefore, a somewhat easier formulation for the objective function is often seen in literature: \( \min_{\gamma(e, x_l, x_h)} w_h + p(e)(w_h - w_l) \) (see e.g. Mas-Colell, Whinston, Green (1995), p. 480.)
induce him to accept the contract. For simplicity, $e$ is measured on the scale of disutility and the reservation utility is set to 0 (but could as well be set to any other reservation utility $U_0$.\textsuperscript{271}

Letting $\mu$ be the Lagrangean multiplier for constraint (1.2) it can be written:

\[
p'\left(e\right)\left(x_h - w_h\right) - p'\left(e\right)\left(x_i - w_i\right) + \\
\mu \left[p'\left(e\right)u\left(w_h\right) - p'\left(e\right)u\left(w_i\right) - 1\right] \\
- p\left(e\right) + \mu p\left(e\right)u'\left(w_h\right) = 0 \\
(p\left(e\right) - 1) + \mu (1 - p\left(e\right))u'\left(w_h\right) = 0
\]  

(1.3) \hspace{2cm} (1.4) \hspace{2cm} (1.5)

Rearranging (1.4) yields:

\[
\mu = \frac{p\left(e\right)}{p\left(e\right)u'\left(w_h\right)} = \frac{1}{u'\left(w_h\right)}
\]  

(1.4)\textsuperscript{'}

Inserting (1.4)\textsuperscript{'} in (1.5) yields:

\[
(p\left(e\right) - 1) - (p\left(e\right) - 1) \frac{u'\left(w_i\right)}{u'\left(w_h\right)} = 0
\]  

(1.5)\textsuperscript{'}

As $p\left(e\right) \neq 1$, it follows from (1.5)\textsuperscript{'}:

\[
\frac{u'\left(w_i\right)}{u'\left(w_h\right)} = 1 \Leftrightarrow u'\left(w_i\right) = u'\left(w_h\right)
\]  

(1.6)

For $u''(\cdot) < 0$ (risk-averseness as assumed above) it can be followed from (1.6) that:

\[
w_i = w_h
\]  

(1.7)

\textsuperscript{271} The reservation utility is the level of utility that the agent demands in order to be willing to work. It can be thought of as the agent's opportunity cost. It is assumed that any gains from trade are appropriated by the principal. This could be justified by assuming that there are more agents than principals, so that the zero profit condition holds for the agents. In this case, this assumption is taken for purely technical reasons and is totally innocuous. It is just one way to derive a Pareto-optimal result.
If effort is contractible, it is optimal for the principal to provide **full insurance**. He pays compensation independent of output. This is an intuitive result: The effort of the agent can be observed and contracted upon. There is **no need to use the wage system to indirectly induce the agent to exert a certain effort**. A contract can stipulate that the agent receives a certain payment if he is exerting high effort and is punished otherwise. Such a contract is called a forcing contract because the principal can force the agent to choose the desired effort level. In this case the **risk-sharing argument comes fully to bear**. As the principal was assumed to be risk neutral and the agent to be risk averse, it is optimal for the principal to assume all the risk\(^{272}\).

**Proposition 1:** If effort is contractible, compensation should not be made contingent on output if it is assumed that the agent is risk-averse and the principal risk-neutral. More generally, if effort is contractible, there is no rationale for output-based schemes in order to avoid shirking. The risk-sharing argument comes fully to bear.

### 2.1.4 Uncontractible Effort

If effort is not contractible, the principal has to offer a contract that maximizes his profit knowing that the agent chooses effort in order to maximize his own utility (i.e. if the agent maximizes utility by choosing the lowest effort level he will do it). If the **principal knows the utility function of the agent**, as was assumed, and if he knows the **probability distribution of outcomes conditional on effort**, the principal can perfectly predict the reaction of the agent to any offered compensation scheme. It will be seen that he has to trade off the benefits of inducing higher effort against the cost of providing incentives (the cost of compensating the agent for risk-taking).

The principal has to solve the following optimization problem:

\[
\max_{(e, w)} \Pi(e, w, w_h) \tag{2.1}
\]

s.t. \( e \in \arg \max_{e'} U(e', w, w_h) \tag{2.2} \)

\[
U(e, w, w_h) \geq 0 \tag{2.3}
\]

\(^{272}\) see The results of Syndicate Theory (Wilson (1968), cit. in Kreps (1990) p. 173f)
The principal's and the agent's pay-off function are the same as in the previous subsection. What is new is condition (2.2). This constraint is the characteristic feature of problems involving hidden action. It reflects the impossibility on the part of the principal to directly influence the agent's effort decision. As the principal cannot observe effort, the agent chooses the effort level that maximizes his own welfare, but the principal anticipates the reaction of the agent. He will choose a compensation scheme that induces the agent to act in the desired way, by providing corresponding incentives. This is why condition (2.2) is called incentive constraint\textsuperscript{273}. The crucial idea behind this argument is rational behaviour: Even if the effort decision is taken autonomously by the agent, his rational behaviour of maximizing expected utility makes him predictable. The principal is effectively choosing a desired effort level which he implements by offering the appropriate compensation scheme. The agent's autonomous decision power is merely adding a constraint. Condition (2.3) is the participation constraint as encountered earlier.

On a technical level, if the action space was finite with n possible actions\textsuperscript{274}, condition (2.2) translates into a set of n incentive constraints\textsuperscript{275}. So, the problem is solved by using the Kuhn-Tucker conditions. If the action space is continuous, as is assumed in this Section, the set of incentive constraints becomes infinite and the described approach becomes analytically intractible\textsuperscript{276}. Therefore, as (2.2) is itself an optimization problem, the first order condition for optimization must hold as long as there is no boundary solution (which was excluded in the assumptions under 2.1.2):

\[
\frac{\partial U(e, w_j, w_h)}{\partial e} = 0 \iff p'(e)[u(w_h) - u(w_j)] - 1 = 0 \quad (2.4)
\]

Some interesting conclusions can be drawn from the analysis of the first order condition: (2.4) only holds if \(u(w_h) - u(w_j) > 0\) as \(p'(e) > 0\). But, \(u(w_h) - u(w_j) > 0\), for \(u'(\cdot) > 0\)\textsuperscript{277} implies \(w_h > w_j\). In addition, for any given level of \(w_j\): \((w_h - w_j) \uparrow \Rightarrow (u(w_h) - u(w_j)) \uparrow \Rightarrow p(e) \downarrow \Rightarrow e \uparrow \forall p''(e) < 0\). In

\textsuperscript{273}It is also sometimes called relative incentive constraint, to underscore that the action to be chosen by the agent must be made relatively more attractive than all other available actions.

\textsuperscript{274}As e.g. in Kreps (1990), p. 577-604.

\textsuperscript{275}The optimal solution is also weakly preferred to itself, which is why there are n and not n-1 constraints.

\textsuperscript{276}see Kreps (1990), p. 605

\textsuperscript{277}It is assumed that more compensation is strictly preferred to less.
words: It can be concluded from the incentive constraint that every optimal compensation scheme in the asymmetric information case involves wage differentials. What is more, the higher these wage differentials, the stronger the incentives (the higher $e$).

Proposition 2: If there is a stochastic relationship between effort and output and effort is not contractible, compensation should be made contingent on output. Incentives rise in strength as compensation differentials increase.

As $U'' = p''(e)(u(w_h) - u(w_l))$ and $p''(e) < 0$ (see assumption under 2.1.2), $U'' < 0$ for all $w_h > w_l$. Thus, the expected utility function $U(\cdot)$ is concave and the first order condition (2.4) is a necessary and sufficient condition for maximization. It can therefore replace the maximization problem of the incentive constraint, considerably simplifying the overall maximization problem:

$$
\max_{w_l, w_h} \Pi(e, w_l, w_h) = p(e)[(x_h - w_h) - (x_l - w_l)] + (x_l - w_l) 
$$

(2.5)

$$
s.t. \quad p'(e)[u(w_h) - u(w_l)] - 1 = 0 
$$

(2.6)

$$
p(e)u(w_h) + (1 - p(e))u(w_l) - e \geq 0 
$$

(2.7)

The participation constraint (2.7) must be binding and therefore holds with equality. Otherwise, the principal could lower compensations $w_l, w_h$ by the same amount, reducing the expected compensation without altering relative incentives (Note that the difference in (2.6) is unaffected.).

Letting $\mu_1$ and $\mu_2$ be the Lagrangean multipliers for (2.6) and (2.7) respectively, it can be written:

$$
p'(e)[(x_h - w_h) - (x_l - w_l)] + \mu_1 p''(e)[u(w_h) - u(w_l)] 
$$

(2.8)

$$
+ \mu_2 [p'(e)(u(w_h) - u(w_l)) - 1] = 0 
$$

$$
p(e) - 1 - u'(w_l)p'(e)\mu_1 + u'(w_l)(1 - p(e))\mu_2 = 0 
$$

(2.9)

---

278 This approach is called the first-order approach (see discussion at the end of the Section). The problem is that the set of incentive constraints of condition (2.2) cannot generally be replaced by the first-order condition. This will only be possible if the agent's v-N-M expected utility function is concave (which was proven above). Generally, concavity is assured if the concavity of the distribution function condition holds, which was guaranteed in the assumptions (see footnote 266 for the two output case). Proving non-decreasing wages in $e$ – as was done in the preceding Paragraph – is also sufficient for the first-order approach to be viable (see Kreps (1990), p. 598 Lemma.).
Risk neutral Agent

In the traditional model it is usually assumed that the principal is risk-neutral and the agent risk-averse. For a moment, however, it is assumed that the agent is risk-neutral. An interesting result can be derived for this case from the two constraints:

Risk-neutrality implies that, at any level of compensation, an increase of expected compensation by 1 unit is valued at exactly 1 unit. Inserting $u'(w_i) = u'(w_h) = 1$ into (2.9) and (2.10) yields:

\[-p(e) + u'(w_h)p'(e)\mu_1 + u'(w_h)p(e)\mu_2 = 0 \tag{2.10}\]

\[-p(e) + 1 - p'(e)\mu_1 + (1 - p(e))\mu_2 = 0 \tag{2.9}'\]

\[-p(e) + p'(e)\mu_1 + p(e)\mu_2 = 0 \tag{2.10}'\]

\[0 - 1 + 0 + \mu_2 = 0 \tag{2.9}'+(2.10)'

\[\mu_2 = 1 \tag{2.11}''\]

Inserting $\mu_2 = 1$ in (2.10) yields:

\[-p(e) + p'(e)\mu_1 + p(e) = 0 \tag{2.11}\]

\[p'(e)\mu_1 = 0\]

As $p'(e) \neq 0$ for $e \in [e_l, e_h]$:

\[\mu_1 = 0 \tag{2.11}''\]

From $\mu_2 = 1$ it can be seen that outcome is efficient. Investing one unit into compensation exactly increases outcome by one unit. Marginal cost equals marginal product. First best can be achieved. From $\mu_1 = 0$ it can be concluded that the incentive constraint is not binding. The principal can therefore achieve the same profit as under symmetric information.

Another interesting result can be derived by setting $\mu_1 = 0$ and $\mu_2 = 1$ into (2.8):

\[p'(e)[(x_h - w_h) - (x_l - w_l)] + p'(e)(u(w_h) - u(w_l) - 1) = 0 \tag{2.8}''\]
Inserting the first order condition (2.4), this expression simplifies to:

\[ p'(e)[(x_h - w_h) - (x_i - w_i)] = 0 \]  

(2.8)''

As \( p'(e) > 0 \) for \( e \in [e_i, e_h) \):

\[ x_h - w_h = x_i - w_i \]  

(2.12)

From (2.12) it can be concluded, that the principal's profit is independent of output. He is fully insured. The agent effectively buys the project from the principal and is left in a residual claimant position. It can easily be seen, that this result also holds for risk-averse principals.

**Proposition 2.1.: In the case of a stochastic production function first best can be achieved if the agent is risk neutral. He becomes residual claimant.**

**Risk-Averse Agent**

It is now assumed that the principal is risk-neutral and the agent risk-averse. An individual is called risk-averse if he values a project below its expected value. If it is assumed that an individual prefers more to less (positive marginal utility), this implies that the individual attributes an ever smaller extra value to an additional unit of the good consumed (diminishing marginal utility). Formally this can be stated by \( u'(\cdot) > 0, \ u''(\cdot) < 0 \).

Unfortunately, in this case the conditions do not simplify. From \( w_h > w_i \) it can be followed that:

\[ u'(w_h) < u'(w_i) \]  

(2.13)

for risk-averse agents \( (u''(\cdot) < 0) \). Solving for the two Lagrangean multipliers yields:

\[ \mu_i = \frac{(1 - p(e))(\mu_2 u'(w_i) - 1)}{u'(w_i) p'(e)} \]  

(2.14)

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279 Rasmusen calls this the “Selling the Shop” Result

280 This assumption was already used above in the case of contractible effort.
Rearranging (2.15) yields:

$$\mu_2 = \frac{u'(w_h) + p(e)[u'(w_i) - u'(w_h)]}{u'(w_h)u'(w_i)}$$  \hspace{1cm} (2.15)

Inserting (2.15)' into (2.14) yields:

$$\mu_2 u'(w_i) = 1 + \frac{p(e)[u'(w_i) - u'(w_h)]}{u'(w_h)}$$  \hspace{1cm} (2.15)'

This expression is positive for all $e^*$, because $p(e) \in (0,1)$, $u'(w_i) > 0$, $u'(w_h) > 0$ (positive marginal utility), $p'(e) > 0$ (see assumptions under 2.1.2) and inequality (2.13). It can easily be seen that expression (2.15) is positive for the same reasons.

If the agent is risk-averse, both the incentive and the participation constraints are binding. The intuition for this result is the following: In order to induce the agent to exert effort there will have to be a wage differential. This implies that the agent is exposed to risk. In order to make the agent’s expected utility from the contract meet his reservation utility level, the risk-averse agent has to be compensated for his risk exposure, driving up expected payments by the principal. Naturally, the principal will not want to pay more than necessary. Therefore, he will devise the wage differential so that it just induces desired behaviour but no more, since this would mean higher risk exposure for the agent and consequently higher cost for the principal. This is why the incentive constraint is binding.

The reason why the participation constraint must be binding has already been given above: If it was not binding it would be possible for the principal to reduce the level of payments to the agent for any observed output by the same amount. This would not affect incentives but would lower cost to the principal. Therefore, this cannot be a property of the optimal incentive scheme.

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281 It is a common assumption in agency theory that if the agent is indifferent about two actions he will choose the one that is preferred by the principal. The reason for this assumption is technical: If it is assumed that the principal sweetens the desired choice just a bit, the tool of optimization will not work as there is no “optimal” sweetener. See Kreps (1990), p. 603f on this point.
The fact that the incentive constraint binds implies that expected cost for the principal will be higher than in the symmetric information or risk-neutrality case. The principal’s expected net benefit will therefore be lower. As the agent’s expected utility remains at the reservation level, over-all welfare is reduced. The outcome is therefore second best\textsuperscript{282}.

\textbf{Proposition 2.2.:} In the case of stochastic production functions and a risk-averse agent, only a second-best solution can be achieved due to the risk-incentive trade-off.

\textbf{Certainty}

In the modeling assumptions (see assumptions under 2.1.2), a stochastic relationship between effort of the agent and output was assumed. Effort was a parameter of the probability distributions of outcome. By choosing his effort level the agent effectively chooses a probability distribution. More specifically, it could not be excluded that output was low although the agent had chosen high effort. If, however, there is a deterministic relationship between output and effort, it is possible to deduce effort from output. All that has to be done is to invert the production function. So, even if effort may not be observable, which is why this case is treated as a case of uncontractible effort, observing output is equivalent to observing input. Therefore, the analysis of effort contractibility applies to this case. This leads to the following proposition:

\textbf{Proposition 3:} In the case of a deterministic production function, output-based schemes achieve first best. A forcing contract can be used.

\textsuperscript{282} Formally, in the case of observable effort the principal, who wants to implement effort level \( e_i \) has to pay the agent: \( w^*(e_i) = u(w^*(e_i)) = u_0 + e_i \) in order to fulfill the participation constraint. In the case of unobservable effort, expected utility of the agent must be: \( E[u(w(x))|e_i] = u_0 + e_i \). As, by Jensen’s inequality, \( u[E(w(x))|e_i] > E[u(w(x))|e_i] \) it follows from the above expression that \( u[E(w(x))|e_i] > u(w^*(e_i)) \). For strictly increasing \( u(\cdot) \) this implies \( E(w(x))|e_i > (w^*(e_i)) \). Therefore, the expected payments from the principal needed to implement any \( e \in [e_1; e_2] \) are lower in the case of observable effort than in the case of non-observable effort. see (Mas-Colell, Whinston, Green (1995), p. 486)
2.1.5 Discussion

The analysis in this Section provided some basic results summarized in the propositions. Most importantly, it was shown that if effort is not contractible, compensation shall be made contingent on output. It was also shown that, in this case, welfare levels can never be higher than in the case of observable effort. Barring the unrealistic cases of a deterministic production function and risk neutrality of both the principal and the agent, the optimal compensation scheme leads to a welfare loss due to imperfect risk sharing.

More general models than the one used in this Section can be set up\(^{283}\) but yield broadly the same results. As this dissertation does not primarily have a technical focus but favours intuitive arguments, the choice of this model seems appropriate.

There is also a different approach to solving the optimization problem than the one used in this Section. This approach was developed by Grossman/Hart (1983)\(^{284}\) and is called the three-step procedure by Fudenberg/Tirole (1990)\(^{285}\). First, one searches for the set of contracts that implement e. Then, among these contracts, the contract which is least costly to the principal is chosen. Finally, one settles for the effort level which maximizes net benefit for the principal\(^{286}\). Often not all of the steps are needed\(^{287}\). Therefore, the approach often allows more simple and elegant analysis\(^{288}\). It also stresses the importance of inducing the agent to choose a certain action and the fact that this inducement is costly. On the other hand, the approach used in this Section is more intuitive to set up.

Another methodological choice was the use of the so-called first-order approach which follows from the assumption of a continuous action space\(^{289}\). This makes it necessary to replace the (infinite) set of incentive constraints with the

\(^{283}\) see Grossman/Hart (1983)

\(^{284}\) Grossman/Hart (1983)

\(^{285}\) see Fudenberg/Tirole (1990)

\(^{286}\) As step 1 and 2 mathematically combine to one (see Rasmusen (1994), p. 176), the approach is also presented in two steps (see Kreps (1990) pp. 587-489 ).

\(^{287}\) e.g. for the results in this Section, steps 1 and 2 would have been sufficient to drive home the results.

\(^{288}\) see footnote (270)

\(^{289}\) Rasmusen (1995), p. 176 wrongly contrasts the first-order approach and the three-step procedure. The first order approach follows from the continuous action space formulation and in this case must also be used in the three-step procedure.
first-order condition for optimization of the agent’s utility. The problem is that this cannot generally be done. It will only be possible if the agent’s v-N-M expected utility function is concave\textsuperscript{290}. Generally, this is only assured if the concavity of the distribution function condition holds, which is not a totally unproblematic assumption. The advantage is that it considerably simplifies the analysis and is also very intuitive in stressing the autonomy of the agent’s decision making if effort cannot be observed. It will therefore be used in many (albeit not all) instances in this thesis. Although Kreps (1990)\textsuperscript{291} considers it as pertaining to the “early literature”, it is still used - possibly for its intuitive appeal\textsuperscript{292}.

\section{Risk-Incentive Trade-off for Linear Contracts}

\subsection{Introduction}

In the following model, specific assumptions are made concerning the production function, the agent’s utility function and the structure of the incentive scheme\textsuperscript{293}. The incentive-risk trade-off can then be explicitly modelled. The following proposition can be derived:

4. If linear contracts are assumed it can be seen that the optimal bonus rate decreases for rising risk-averseness, rising project risk and rising curvature of the disutility function. In the case of a deterministic production function ($\sigma^2 = 0$), all the risk is assumed by the agent ($b=1$). The forcing contract was excluded by the linear sharing rule, but it can be seen that also a linear contract can achieve first best.

\subsection{Modeling Assumptions}

The project production function is assumed to be linear and disturbed by an error term $\varepsilon$, where $y$ is output, $a$ is effort, measured on the scale of expected project value\textsuperscript{294}, and $\varepsilon$ is a random variable, which is normally distributed with zero mean.

\footnotesize

\begin{enumerate}
\item see e.g. Kreps (1990), pp. 605ff. and the literature on the first order approach: e.g. Rogerson (1985)
\item see Kreps (1999), p. 604
\item see Bester (2001)
\item see e.g. Gibbons (2001)
\item This is important, because otherwise it would be difficult to interpret negative values for a.
\end{enumerate}
and variance $\sigma^2$. Thus, the agent controls the mean of a normally distributed random variable by his choice of effort$^{295}$:

$$y = a + \varepsilon, \quad \varepsilon \sim N \left(0, \sigma^2\right).$$

(3.1)

Note that in this model not only the set of possible actions but also the range of outcomes is continuous.

The incentive scheme is linear with a bonus rate $b$ and a base salary $s$ $^{296}$:

$$w(y) = s + by.$$  

(3.2)

The agent's utility function is an exponential $v$-$N$-$M$ utility function$^{297}$:

$$U(x) = 1 - e^{-rx}.$$  

(3.3)

Remark: It can easily be seen that $r$ is a measure of risk-averseness$^{298}$:

$$- \frac{U''}{U'} = r.$$  

(3.4)

$^{295}$ This is the state-space formulation of the Agency Problem. see Spence/Zeckhauser (1971). Obviously, the intuition is that higher effort leads to higher mean and with the variance unchanged to a more favourable distribution of outcomes. However, as Stiglitz, Rothschild (1970) have shown, there are examples where a lottery with lower mean and higher variance is preferred by a risk-averse agent. The reason is, that $\mu, \sigma$-analysis is not generally equivalent to the concept of first/order stochastic dominance. Yet, in the case of a normally distributed error term, it is easy to see that higher effort levels are equivalent to choosing stochastically higher distributions of output.

$^{296}$ Linearity is, prima facie, a totally arbitrary assumption. In Sub-Section IV2.4.3 below, reasons are given why it might be justified to constrain the shape of the optimal sharing rule to be linear.

$^{297}$ The choice of an exponential Bernoulli function implies that the agent's absolute risk-averseness is constant in wealth $x$ and his relative risk averseness increases in wealth. Yet, it is often natural to assume decreasing absolute risk-averseness and non-increasing relative risk-averseness. Otherwise the agent would e.g. only be ready to invest a constant absolute amount of his wealth into a risky asset. This implies that the share of his wealth invested in the risky asset decreases in wealth.

$^{298}$ This shows that the exponential utility function can be fully recovered from the coefficient of absolute risk averseness (also called Arrow/Pratt measure).
The agent’s disutility of effort is given by \( c(a) \) and is assumed to be strictly increasing and convex in \( a \):

\[
c(a) > 0, \quad c'(a) > 0, \quad c''(a) > 0.
\] (3.5)

The principal is risk neutral, which means that he values a project at its expected value. His certainty equivalent \( CE \) is therefore:

\[
CE = V[E(x)] = E(x).
\] (3.6)

### 2.2.3 The Model

The principal’s pay-off for a contract \( \gamma = (b,s) \) is:

\[
\Pi(\gamma,a) = (1-b)(a + \varepsilon) - s
\] (3.7)

The agent’s pay-off is:

\[
w(a,\gamma,c(\cdot)) = s + b(a + \varepsilon) - c(a).
\] (3.8)

The principal’s optimization problem is:

\[
\max_{\gamma} \Pi(\gamma,a^*) = (1-b)a^* - s
\] (3.9)

\[
\text{s.t. } a^* \in \arg\max_{a'} CE(a',\gamma,\sigma,c(\cdot)) \geq 0
\] (3.10)

\[
CE(a,\gamma,\sigma,c(\cdot)) \geq 0
\] (3.11)

The principal designs the contract in order to maximize expected profit. As he cannot observe the agent’s choice of effort, the agent will choose the level of effort which will maximize his own expected utility. As it is assumed that the principal knows the agent’s utility function he will perfectly predict the agent’s reaction to a given contract. So, one can think of the principal’s problem as designing a contract, keeping in mind that the agent will always optimally adjust his effort (see above). For technical reasons the problem is stated in certainty equivalent and not in expected utility terms.
First the maximization problem of the constraint set is replaced by the first-order condition which can be shown to be necessary and sufficient.

It is a well-known result for an exponential utility function and normally distributed output that the certainty equivalent can be written as:

\[ CE = \bar{U} \left( E \left( U(a, \gamma, c(\cdot)) \right) \right) = s + ba - c(a) - \frac{rb^2 \sigma^2}{2}. \quad (3.12) \]

The first-order condition is:

\[ \frac{\partial CE}{\partial a} = b - c'(a^*) = 0 \Rightarrow c'(a^*) = b, \quad (3.13) \]

which means that for any incentive scheme the agent maximizes utility by choosing the effort level which sets his marginal disutility equal to the bonus rate. The second-order condition for a maximum is:

\[ \frac{\partial^2 CE}{\partial^2 a} = -c''(a^*) < 0. \quad (3.14) \]

(3.14) always holds because of the convexity of the disutility function. Therefore (3.12) is convex and the first-order condition (3.13) is necessary and sufficient.

Restating the maximization problem, it can be written:

\[
\begin{align*}
\max_{a^*, b} & \ (1 - b)a^* - s \\
\text{s.t.} & \ c'(a^*) = b \Rightarrow a^* = c'(b)^{-1} = a^*(b) \\
\end{align*}
\]

\[ s + ba - c(a) - \frac{rb^2 \sigma^2}{2} \geq 0. \quad (3.11) \]

299 The first order approach once again is needed because of the continuous action space.

300 See appendix to this Chapter.

301 The range of \(a\) is \(R\). So one does not have to bother about boundary solutions. It is also assumed that the principal has to offer a contract even if his expected pay-off is negative and cannot set the terms of the contract so that the agent surely rejects it. Otherwise, there would be a problem if cost increase is to steep.
Remark: It can easily be seen from (3.10)' that $b$ is positive as $c'(\cdot) > 0$. It can also be said that optimal effort is positive, $a^* > 0$. This is because $c'(\cdot) > 0$ implies $c'(\cdot)^{-1} > 0$.

Inserting (3.10)' into (3.11)' gives:

$$\max_{s,b} (1-b)a^*(b) - s$$

$$s.t. s + ba^*(b) - c\left(a^*(b)\right) - \frac{rb^2\sigma^2}{2} = 0.$$ 

In addition, the principal will not offer more compensation than is needed to assure the participation of the agent. Therefore, the participation constraint will be binding. Letting $\mu$ be the Lagrangean multiplier for (3.11)'' yields:

$$\frac{\partial L}{\partial b} = -a^*(b) + (1-b)a^*(b) +$$

$$\mu (\alpha'(b) + ba^*(b) - c'(a^*(b))a^*(b)) - r\sigma^2b = 0$$

$$\frac{\partial L}{\partial s} = -1 + \mu = 0 \Rightarrow \mu = 1.$$ 

Inserting (3.16) in (3.15):

$$a^* - c'(a^*)a^* - r\sigma^2b = 0.$$ 

Inserting (3.10)' in (3.17) and rearranging yields:

$$b = \frac{a^*}{a^* + r\sigma^2}.$$ 

Differentiating (3.10)' and solving for $a$ shows that the marginal impact of a higher bonus rate on the level of optimal effort is lower the more convex the disutility function:

---

302 As was argued above, it is possible to reduce expected cost of the principal without changing the incentives if the participation constraint is not binding.
\[
\frac{\partial}{\partial b} c'(a^*(b)) = \frac{\partial}{\partial b} b \\
c''(a^*)a^{*'} = 1 \\
a^{*'} = \frac{1}{c''}.
\]

Inserting (3.19) in (3.18) gives the optimal bonus rate\textsuperscript{303}:

\[
b = \frac{1}{1 + r \sigma^2 c''}.
\]

Proposition 4.: If linear contracts are assumed it can be seen that the optimal bonus rate decreases for rising risk-averseness, rising project risk and rising curvature of the disutility function. In the case of a deterministic production function \(\sigma^2 = 0\), all the risk is assumed by the agent \(b=1\). The forcing contract was excluded by the linear sharing rule, but it can be seen that also a linear contract can achieve first best.

Total welfare can be calculated by adding the certainty equivalents of the agent and the principal. Inserting (3.7) into (3.6) and adding with (3.12) gives total surplus in the case of private information:

\[
a - s - b(a) + s + ba - c(a) - \frac{rb^2 \sigma^2}{2} \\
= a - c(a) - \frac{rb^2 \sigma^2}{2}.
\]

In the perfect information case, gains of trade are:

\[
a - c(a).
\]

Subtracting (3.22) from (3.21) gives the welfare loss in the case of private information compared to symmetric information holding the effort level constant:

\textsuperscript{303} The same result can be obtained maximizing the joint welfare function of the two parties. This is mathematically more straightforward but less intuitive.
One should note that total welfare loss will be higher. In the above case, effort levels were held constant. In fact, optimal effort levels in the symmetric information case will be higher\textsuperscript{304} but the point here was to show that in the asymmetric information case a welfare loss occurs.

### 2.2.4 Discussion

The main appeal of the model of this subsection is the existence of a closed-form solution which negatively links the use of variable fee contracts to the agent's level of risk aversion, the level of project risk and the convexity of the disutility function. The weakness of this model is its lack of generality and its unrealistic assumptions as e.g. the constant absolute risk-averseness of the agent's preferences. Especially the linear sharing rule seems to be completely arbitrary. Although an argument in rescue of linear sharing rules will be presented in a later Chapter (see 2.4.3), the main reason why this model was presented is its intuitive appeal and its suitability as a starting point for further analysis.

### 2.2.5 Appendix

Ad (3.12):

The explicit expression for the following equation is needed:

\[ CE = \bar{U}(E(U(a, \gamma, c(\cdot)))) \].

Inserting the utility function, one can write for expected utility:

\[ \frac{rb^2 \sigma^2}{2} \].

\textsuperscript{304} It is not necessarily the case, that optimal effort levels are lower than in the first best case. This is because there are two sources of loss in the second best equilibrium. One is divergence from the efficient effort level. The other is the cost of separation of different possible actions. These are low if the statistical signal is very strong. Then, it is easy to infer from a certain output level which action was chosen. As the chance of error is lower, and consequently the risk of being innocently punished, the risk premium is lower. Kreps (1990), p. 602f) constructs a simple example to illustrate this point. This will not be the case here, as there is no discontinuity of the likelihood ratios in the model discussed in this Sub-Section.
\[ E \left( 1 - e^{-r(ba + e)s - c(a)} \right) = 1 - e^{-r[b(a + e)s - c(a)]} E \left( e^{-rbe} \right). \] (3.25)

\( E \left( e^{-rbe} \right) \) can be written as:
\[
E \left( e^{-rbe} \right) = \int e^{-rbe} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2}\frac{x^2}{\sigma}} \, dx = \frac{1}{\sigma \sqrt{2\pi}} \int e^{-rbe} \frac{1}{2\sigma^2} \, dx.
\] (3.26)

A known rule of integration\(^{305}\) is:
\[
\int e^{-rbe} \, dx = e^{\frac{q^2}{4p}}. \] (3.27)

Setting \( p = \frac{1}{2\sigma^2} \) and \( q = -rb \) into (3.27) yields:
\[
E \left( e^{-rbe} \right) = \frac{1}{\sigma \sqrt{2\pi}} \int e^{-rbe} \frac{1}{\sigma^2} \, dx = \frac{1}{\sigma \sqrt{2\pi}} e^{\frac{r^2b^2}{2\sigma^2}} = e^{\frac{r^2b^2}{2\sigma^2}}. \] (3.28)

Inserting this result into equation (3.25) yields:
\[
E \left( U \right) = 1 - e^{-r[b(a + e)s - c(a)] + \frac{r^2b^2}{2\sigma^2}}. \] (3.29)

The certainty equivalent is defined as:
\[
CE = \overline{E} \left[ E \left( U \right) \right]. \] (3.30)

Inverting \( U(x) : y = 1 - e^{-rx} \) yields:
\[
\overline{U} (x) : x = 1 - e^{-ry}. \]

\(^{305}\) Rysik, Gradstejn (1965)
Solving for $y$ gives:
\[
\bar{U}(x) : y = \frac{\ln(1-x)}{-r}.
\] (3.31)

Setting $x = E(U)$ in (3.31) gives the certainty equivalent:
\[
CE = \frac{\ln(1-E(U))}{-r}.
\] (3.32)

Inserting (3.29) into (3.32) yields:
\[
CE = \frac{\ln\left(1 - 1 + e^{-r(ba + s - c(a)) + \frac{rb^2 \sigma^2}{2}}\right)}{-r}
= s + ba - c(a) - \frac{rb^2 \sigma^2}{2}. qued.
\] (3.33)

### 2.3 Risk Sharing

#### 2.3.1 Introduction

It was argued above that variable fees may provide useful incentives in situations of hidden action, but also create imperfect risk sharing. Quite apart from any other consideration, a closer look at the mechanics of risk sharing is warranted: In the traditional agency models, the principal is always assumed to be risk-neutral, while the agent is assumed to be risk-averse or risk-neutral.

This need not be the case. The assumption that the principal is risk-neutral is often justified by the argument that he is the economically more potent party to the contract. This is largely inspired by the traditional story behind principal-agency models referring to the relationship between a company and its employees. There are several arguments for why the economically more potent party should be less risk-averse: First, it is often plausible to assume that as a person becomes wealthier his absolute level of risk aversion decreases. Second, if the company is held by an entrepreneur, he might be the less risk-averse type of person in the first place. In addition, employees usually only work for one company while the owner might hold many different companies. So, he is probably better diversified. This last point is especially true for publicly held companies. However, in the case of a
client and his consultant, things can be different. The small consultant partnership is clearly more risk-averse than its multinational client, but this changes if the big international consultancy firm provides services to a small start-up company through its incubator branch. Clearly, in the setting analysed so far - the case where, say, the principal is risk-averse and the agent risk-neutral - does not appear to be problematic as optimal incentive provision and optimal risk sharing are compatible. But what if there is a bilateral moral hazard problem? To answer this and other questions, one should look at risk sharing in its own right.

In the following, two separate sources of value creation by risk sharing shall be explored: differences in risk attitudes which might arise from predisposition or different levels of wealth and differences in diversification. This will be modelled in a linear-normal-exponential setting and the following propositions are derived:

5. Without Diversification:

5.1. If both parties are risk-averse, it will always be optimal to engage in some degree of risk sharing. Each party’s share in the risky part of their pay-off equals the ratio of their risk tolerance and overall risk tolerance. The optimal bonus rate thus increases as the agent becomes more risk-tolerant relative to the principal. It does not depend on any specific distributional assumptions.

5.2. If one party is risk-neutral while the other party is risk-averse, it is optimal for the risk-neutral party to assume all the risk. If both parties are risk-neutral, the choice of the bonus rate does not matter for risk sharing purposes.

306 Technically, in line with the usual instrumentalist flexibility in creating models, it is common to use simple exponential Bernoulli functions, although they exhibit constant absolute risk aversion, even if it is assumed that there is decreasing absolute risk aversion. This is not as big a problem as it seems: In order to model decreasing absolute risk aversion, one is just doing as if the wealthy party was less risk-averse by nature and then uses the exponential Bernoulli function to approximate local preferences over lotteries. The same can be done to accommodate for diversification effects. In the case of the publicly held company, shareholders are not actually risk-neutral, but they are diversified. So, collectively, it is in their interest that the company is managed as if held by a risk-neutral individual (in the absence of financial distress costs). In this Section, however, diversification effects will be explicitly modelled to highlight the fact that the more risk-averse company might not necessarily be less diversified and vice versa (Take the example of a private-equity partnership).
5.3. It can be seen that the advantage of variable fees over flat fees increases for rising project risk, rising absolute levels of risk averseness and rising relative risk tolerance of the agent and vice versa.

6. With Diversification

6.1. The optimal sharing rule for risk sharing with diversification can be written as the sum of the optimal sharing rule without diversification and a correction term, accounting for the portfolio effects.

6.2. The portfolio effect will increase the optimal bonus rate the stronger the diversification effect in the portfolio of the agent is relative to the principal’s diversification effect and vice versa.

6.3. As risk tolerance of the agent relative to the principal increases, the optimal bonus rate always becomes higher if the coefficients of correlation are positive. If adding the project actually lowers portfolio risk of the principal, the optimal bonus rate will possibly decrease for increasing risk tolerance of the agent relative to the principal.

6.4. If the difference between optimal risk sharing and the flat fee case is taken as an indicator for the importance of risk sharing, importance increases for increasing project risk, lower absolute levels of risk tolerance of the parties involved and increasing differentials of “specific risk appetite” which depends on both the relative risk tolerance and the specific way the project interacts with the parties’ portfolios.

2.3.2 The Model

In this first subsection, only differences in risk attitudes are considered. Diversification will enter the analysis in the next subsection as an extension to this basic model. Variable fees create value by improving risk sharing if the sum of certainty equivalents of the principal and the agent in the case of flat fees \((CE_p + CE_A)\) is lower than in the case of variable fees \((CE'_p + CE'_A)\):

\[
CE_p + CE_A < CE'_p + CE'_A. \tag{4.1}
\]
An alternative way to think of this condition is that value is created if by moving from flat fees to variable fees the certainty equivalent of the agent \((CE_A)\) is decreasing less than the certainty equivalent of the principal \((CE_P)\) increases:

\[
CE_A' - CE_A < CE_P' - CE_P. \tag{4.1}
\]

The optimal degree of risk sharing is attained if the sum of certainty equivalents is maximized:

\[
\max_b \left( CE_P' + CE_A' \right), \tag{4.2}
\]

where \(b\) represents the bonus rate of the linear incentive contract.

Staying in the above framework of exponential utility functions and normally distributed outcomes, the certainty equivalents for variable fee contracts can be calculated as follows:

\[
\begin{align*}
CE_P' &= \bar{x}_p - \left(1 - b\right)^2 \frac{r_p \sigma^2}{2} \\
CE_A' &= \bar{x}_A - b^2 \frac{r_A \sigma^2}{2}.
\end{align*} \tag{4.3}
\]

Where \(\bar{x}_p, \bar{x}_A\) is the expected value, \(r_p, r_A\) is the coefficient of risk averseness for the principal and the agent respectively, and \(b\) is the bonus rate. (By setting \(b = 0\) it becomes obvious that this formulation comprises the case of flat fees.)

Inserting (4.3) into (4.2) gives:

\[
b^* \in \arg \max_b \bar{x}_p + \bar{x}_A - \left(1 - b\right)^2 \frac{r_p \sigma^2}{2} - b^2 \frac{r_A \sigma^2}{2}. \tag{4.4}
\]

As \(\bar{x}_p, \bar{x}_A\) do not depend on \(b\), this can be reformulated as:

\[
b^* \in \arg \min_b \left(1 - b\right)^2 \frac{r_p \sigma^2}{2} + b^2 \frac{r_A \sigma^2}{2}. \tag{4.5}
\]
So, the problem can be thought of as choosing $b$ in order to minimize the risk premium. Differentiating (4.5) yields the first-order condition:

$$b^* \sigma^2 (r_p + r_A) - r_p \sigma^2 = 0$$  \hspace{1cm} (4.6)

**Both parties are strictly risk-averse**

As both parties are strictly risk-averse and the project is risky $\sigma^2 (r_p + r_A) > 0$. The objective function is therefore convex and the first-order condition is necessary and sufficient for a global minimum. Solving (4.6) for $b^*$, it can be written:

$$b^* = \frac{r_p}{r_p + r_A} = \left(1 + \frac{r_A}{r_p}\right)^{-1}.$$  \hspace{1cm} (4.6)'

In the literature, this expression can also be found in the form of:

$$b^* = \frac{\tau_A}{\tau_A + \tau_p} = \left(1 + \frac{\tau_p}{\tau_A}\right)^{-1},$$  \hspace{1cm} (4.6)''

where $\tau_i = 1/r_i$ and is called the coefficient of risk tolerance$^{307}$. As this is the more intuitive concept, propositions will be derived in terms of risk tolerance.

Interpreting condition (4.6)'' yields the intuitive result that the optimal bonus rate increases as the agent becomes more risk-tolerant relative to the principal.

$$\left(\frac{\tau_p}{\tau_A} \downarrow\right) \Rightarrow \left(b^* \uparrow\right)$$  \hspace{1cm} (4.7)

In addition, perhaps less intuitively, the optimal bonus rate which reflects the agent’s share in the risky part of his compensation equals the ratio of his risk tolerance and the over-all risk tolerance of both parties. Also, note that distributional assumptions play no part in the optimal sharing rule.

---

$^{307}$ see Kreps (1990), p. 173
It can also be seen that if both parties are strictly risk-averse it will always be optimal to engage in some degree of risk sharing:

\[ 0 < b^* < 1. \] (4.8)

One could easily think otherwise: If one party is more risk-tolerant than the other, it will suffer less disutility for taking risk than the other party. So, it seems plausible that it should shoulder all the risk. But this argument is flawed because the \textit{risk premium is convex in} \( b \). Of course, there are situations where \( b^* \) is close to zero (for very high \( \tau_p / \tau_A \)) or close to one (for very low \( \tau_p / \tau_A \)).

\[ \lim_{\tau_p / \tau_A \to 0} b^* = 0, \quad \lim_{\tau_p / \tau_A \to \infty} b^* = 1. \] (4.9)

**Proposition 5.1**: If both parties are risk-averse, it will always be optimal to engage in some degree of risk sharing. Each party’s share in the risky part of their pay-off equals the ratio of their risk tolerance and over-all risk tolerance. The optimal bonus rate thus increases as the agent becomes more risk-tolerant relative to the principal. It does not depend on any specific distributional assumptions.

**Principal risk-neutral, agent strictly risk-averse and vice versa**

Setting \( r_p = 0 \) the first-order condition in (4.6) becomes:

\[ b^* r_A \sigma^2 = 0, \] (4.10)

which means that \( b^* = 0 \).

Similarly, if the agent is risk-neutral and the principal riskaverse, one gets:

\[ (b^* - 1) r_p \sigma^2 = 0 \] (4.11)

which can only hold true if \( b^* = 1 \).

Thus, in line with intuition, the risk-neutral party, whether it is the principal or the agent, will assume all the risk. This is the result, underlying the result in subsection (2.1.4) which showed that first-best can be achieved in the case of a risk-neutral agent, even if effort is not observable. The effect of the agent’s choice of effort can then be fully internalized, without creating inefficient risk sharing.
Both parties risk-neutral

If both parties are risk-neutral, the sum of certainty equivalents is always the same independent of \( b \). The choice of \( b \) does not matter for risk sharing purposes.

Proposition 5.2.: If one party is risk-neutral, while the other party is risk-averse it is optimal for the risk-neutral party to assume all the risk. If both parties are risk-neutral, the choice of the bonus rate does not matter for risk sharing purposes.

Importance of risk sharing

As an indicator for the importance of finding the optimal sharing rule, the difference between optimal risk sharing and the flat fee case is calculated:

\[
\frac{r_p^2 \sigma^2}{2(r_p + r_A)} = \left(1 + \frac{\tau_p}{\tau_A}\right)^{-1} \frac{r_p \sigma^2}{2}
\]

(4.12)

Proposition 5.2.: It can be seen that the advantage of variable fees over flat fees increases for rising project risk \((\sigma^2 \uparrow)\), rising absolute levels of risk averseness \((r_p \uparrow)\) and rising relative risk tolerance of the agent \((\tau_p/\tau_A \downarrow)\) and vice versa.

2.3.3 Model Extension: Diversification

In this subsection, in an extension to the basic model developed above, the effects of different levels of diversification is studied. The crucial idea is that the relevant risk of a project to a party depends on the risk the project adds to this party’s portfolio. This might be well below its stand-alone risk and might differ among parties.

The basic method remains the same: First, the risk premium is calculated dependant on the bonus rate \( b \). Then, the optimal bonus rate \( b^* \) is derived using the first-order condition and checking if it is necessary and sufficient. Finally, as an indicator of the importance of risk sharing, the difference between optimal risk sharing and the flat fee case is calculated. The idea is that risk-sharing arguments should be given more weight if the potential value creation is higher. Interpreting the result, one can show which variables are driving the optimal bonus rate and the importance of risk sharing arguments. It will be seen that the results of the simple model above are special cases of this more general model.
As in the above case, the sum of certainty equivalents is maximized:

$$\max_b (CE'_p + CE'_A).$$  \hfill (4.13)

The certainty equivalents now reflecting diversification can be calculated as follows (Note that this formulation comprises the flat fee case for $b = 0$):

$$CE'_p = \bar{x}_{p \pi} + (1 - b)x - s - \frac{r_p \left( \sigma_{\pi p}^2 + (1-b)^2 \sigma^2 + 2(1-b)d_p \right)}{2}$$  \hfill (4.14)

$$CE'_A = \bar{x}_{\pi \alpha} + bx + s - \frac{r_A \left( \sigma_{\pi A}^2 + b^2 \sigma^2 + 2bd_A \right)}{2}$$

where

$$d_p = \text{cov} \left( \bar{X}_{\pi p}, \bar{X} \right) = \rho_p \sigma_{\pi p} \sigma_p$$ \hfill (4.15)

and $\rho_p$ is the coefficient of correlation, $x_{\pi p}$ is expected value and $\sigma_{\pi p}$ is variance of the portfolio of the principal and $d_A, \rho_A, x_{\pi \alpha}, \sigma_{\pi \alpha}$ defined accordingly for the agent.

Inserting (4.14) into (4.13) yields:

$$b^* \in \arg \max_b \bar{x}_{\pi p} + \bar{x}_{\pi A} - \frac{r_p \left[ \sigma_{\pi p}^2 + (1-b)^2 \sigma^2 + 2(1-b)d_p \right]}{2}$$

$$- \frac{r_A \left[ \sigma_{\pi A}^2 + b^2 \sigma^2 + 2bd_A \right]}{2}$$  \hfill (4.16)

Reformulating (4.16) yields:

$$b^* \in \arg \min_b \frac{r_p \left[ \sigma_{\pi p}^2 + (1-b)^2 \sigma^2 + 2(1-b)d_p \right]}{2}$$

$$+ \frac{r_A \left[ \sigma_{\pi A}^2 + b^2 \sigma^2 + 2bd_A \right]}{2}$$  \hfill (4.17)
For $b^*$ the first order condition must hold:

$$b^* \sigma^2 (r_p + r_A) - r_p \sigma^2 - r_p d_p + r_A d_A = 0 \quad (4.18)$$

**Both parties are strictly risk-averse**

Checking the second order condition $\left( \sigma^2 (r_p + r_A) > 0 \right)$, it is found that the objective function is concave in $b$ and therefore $b^*$ is a global minimum.

Rearranging, one can write for $b^*$:

$$b^* = \frac{r_p \sigma^2 + r_p d_p - r_A d_A}{\sigma^2 (r_p + r_A)} = \frac{r_p + r_A}{\sigma^2 (r_p + r_A)} \quad (4.19)$$

The above case (without diversification) can of course be shown to be a special case of this more general version by setting $d_p = d_A = 0$.

Setting $d_i = \rho_i \sigma_i \sigma$ (see (4.15)), $\vartheta_i = -\rho_i \sigma_i$, $r_i = 1/\tau_i$ and doing some tedious algebra (4.19) can be written as:

$$b^* = \left(1 + \frac{\tau_p}{\tau_A}\right)^{-1} + \frac{1}{\sigma} \left[ \left(1 + \frac{\tau_A}{\tau_p}\right)^{-1} \vartheta_A - \left(1 + \frac{\tau_p}{\tau_A}\right)^{-1} \vartheta_p \right]. \quad (4.20)$$

$\tau_i$ is the party’s risk tolerance as above. $\vartheta_i$ can be interpreted as follows: $\rho_i \sigma_i$ can be seen as a factor determining the project’s contribution to the principal’s overall risk. If the coefficient of correlation is 1, the project’s variance is simply added to the portfolio’s variance in order to get the combined risk. If it is less than 1, the combined risk is lower than the sum of variances. In extreme cases (if $\rho_i < 0$), the combined effect can actually lower overall portfolio risk. Generally speaking, the lower this expression the stronger the diversification effect. Therefore, $\vartheta_i = -\rho_i \sigma_i$ can be interpreted as an indicator of the strength of the diversification effect. (Note that the value of $\vartheta_i$ is negative as long as the coefficient of correlation $\rho_i$ is positive.)

The effect of diversification can be studied in a first approach by setting $\tau = \tau_p = \tau_A$:

$$b^* = \frac{1}{2} + \frac{(\vartheta_A - \vartheta_p)}{2\sigma} \quad (4.21)$$
It can be seen that the optimal bonus rate is higher if the diversification effect is stronger for the agent than for the principal. Project risk $\sigma$ can be interpreted as a scale variable of the correction term: The higher the absolute risk, the lower the effect on the bonus rate as small changes already make a big difference.

The assumption of equal risk averseness has been instructive. Now, the more general case will be considered. Rewriting and interpreting (4.20),

$$b^* = \left(1 + \frac{\tau_p}{\tau_A}\right)^{-1} + \frac{1}{\sigma} \left(1 + \frac{\tau_A}{\tau_p}\right)^{-1} \vartheta_A - \left(1 + \frac{\tau_p}{\tau_A}\right)^{-1} \vartheta_p$$

(4.20)

the following propositions can be derived:

**Proposition 6.1.** The optimal sharing rule for risk sharing with diversification can be written as the sum of the optimal sharing rule without diversification and a correction term, accounting for the portfolio effects.

**Proposition 6.2.** The portfolio effect will increase the optimal bonus rate the stronger the diversification effect in the portfolio of the agent is relative to the principal’s diversification effect and vice versa.

As the agent becomes more risk-tolerant relative to the principal ($\tau_p/\tau_A$ decreases), the first term on the right hand side of equation (4.20) increases. Analysing the second term is more difficult:

If both coefficients of correlation are positive, $\vartheta_A, \vartheta_p$ will be negative. So, decreasing $\tau_p/\tau_A$ will put less weight on the negative and more weight on the positive part of the parenthesized expression. Therefore, the second term will also rise.

If the diversification effect of the agent is so strong that $\vartheta_A$ turns positive, then decreasing $\tau_p/\tau_A$ will put less weight on a positive and less weight on another positive part of the parenthesized expression. So, the parenthesized expression will roughly stay the same. As the agent becomes more risk-tolerant he does not appreciate as much that he can actually reduce portfolio risk by accepting the project, but terms 1 and 2 combined will most likely rise.

If the diversification effect of the principal is so strong that $\vartheta_A$ turns positive, then decreasing $\tau_p/\tau_A$ because the principal becomes less risk-tolerant will actually put more weight on the negative part of the parenthesized expression.
So, it definitely decreases and even turns negative. Therefore, the combined effect of both terms will be low and can even turn negative. In extreme cases, \( b \) could even turn negative.

**Proposition 6.3.:** As the risk tolerance of the agent relative to the principal increases, the optimal bonus rate always becomes higher if the coefficients of correlation are positive. If adding the project actually lowers the portfolio risk of the principal, the optimal bonus rate will possibly decrease for increasing the risk tolerance of the agent relative to the principal.

In this last special case, the portfolio effect would not only reduce the risk contribution of the project below its stand-alone risk, but make the risk contribution negative so that the portfolio of the agent is absolutely more risky before adding the project. The intuition is that the higher the risk averseness of the principal, the more attractive it is for him to add the project to his existing portfolio for its risk-reducing properties. Normally, correlations among projects and the portfolio will be positive, and the only question is if portfolio effects reinforce, mitigate or reverse the general direction reflected by different tolerance of risk.

It is convenient to think of the issues raised here in terms of differentials of “specific risk appetite” between parties. It is specific because it is the appetite for the specific risk of the particular project in question, and not just any kind of risk. It depends on two factors: First, the parties’ risk tolerance and second, the diversification effects in the parties’ portfolio. One has to be careful to understand how these factors interact. Usually, ceteris paribus the “specific risk appetite” of a party increases as its risk tolerance increases. If, however, diversification effects are so strong that they turn negative, the opposite is the case. The lower risk tolerance, the higher will be the party’s “specific risk appetite”. This is the case where the project’s contribution is not only lower than its stand-alone value but actually negative, i.e. the overall risk of the party’s portfolio is decreased by adding the project\(^{308} \). So, in the case of diversification, situations may be imagined where the optimal bonus rate is 0 or 1, even if none of the parties is risk-neutral. It can even be imagined that it is lower than 0 or higher than 1. This suggests that

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\(^{308}\) If a savings and loan bank has issued bonds with a fixed coupon and subsequently interest rates go down, the margins of the bank will shrink. At the same time, a property dealer will benefit from the lower interest rates as people will get cheaper financing or switch assets from bonds into property. So, the consultant could counterbalance the risk of doing a variable fee project with a savings and loan bank by acquiring a variable fee project with a property dealer. Usually, however, the consultant will not agree to performance measures that do not take account of such factors.
the parties can gain by swapping payments contingent on project outcome (providing insurance for other risks the counterparty holds)\textsuperscript{309}.

**Principal risk-neutral, agent strictly risk-averse and vice versa**

Setting $r_p = 0$ into the first-order condition (4.18) yields:

\[ b^* \sigma^2 r_A + r_A d_A = 0. \]  

(4.22)

Solving for $b^*$ and rearranging gives:

\[ b^* = \frac{\vartheta_A}{\sigma}. \]  

(4.23)

So, for positive coefficients of correlation $b^*$ becomes negative. This does not look very realistic. Often $b$ will be bounded to be non-negative, but the following interpretation can be given: If the agent is risk-averse and the risk of the project positively correlates with his portfolio, the parties might be interested to arrange an insurance contract wherein the agent pays the principal to carry not only the risk of the project, but also to provide further coverage for other risks the agent faces.

If the principal is strictly risk-averse and the agent risk-neutral, $b^*$ can be written as:

\[ b^* = 1 + \frac{\vartheta_p}{\sigma} \]  

(4.24)

Analogous to above this time the principal is interested to buy insurance from the agent if the coefficient of correlation is positive.

**Importance of risk sharing**

The difference between optimal risk sharing and flat fees is given by:

\[ \text{Importance of risk sharing} \]

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\textsuperscript{309} This seems very contrived. A company that wishes to do away with certain risks will probably more likely resort to an insurance policy or a financial engineering product.
\[ \frac{(r_p \sigma^2 + r_p d_p - r_A d_A)^2}{2 \sigma^2 (r_p + r_A)} \]  

(4.25)

Inserting \( d_K = d_B = 0 \), gives:

\[ \frac{r_p \sigma^2}{2 (r_p + r_A)} \]  

(4.26)

which was exactly the result above in the model without diversification.

Inserting \( d_i = -\delta \sigma \) and \( r_i = 1/\tau_i \) into (4.25) rearranging gives:

\[ \frac{\tau_A \sigma + (\tau_p \delta_A - \tau_A \delta_p)}{2 \tau_A \tau_p (\tau_A - \tau_p)} \]  

(4.27)

Proposition 6.4.: If the difference between optimal risk sharing and the flat fee case is taken as an indicator for the importance of risk sharing, importance increases for increasing project risk, lower absolute levels of risk tolerance of the parties involved and increasing differentials of "specific risk appetite", which depend on both the relative risk tolerance and the specific way the project interacts with the parties' portfolios.

2.3.4 Discussion

This Section inquired into what is driving perfect risk sharing and highlighted the parties' level of risk tolerance (both absolute and relative to each other), the specific quality of the risk involved as determined by its correlation with existing risk-exposure and the level of project risk.

Part of the results of the first subsection (which excluded portfolio effects) reveals itself to be a special case for 2 individuals of a more general result of syndicate theory\textsuperscript{310}: The fact that if a risk-neutral party is involved it carries all the risk; the independence of the sharing rule in each state of the probability assigned to that state; and the fact that the risky part of each party's compensation is

\textsuperscript{310} Syndicate theory is an application of a methodology in the theory of social choice which solves the social choice problem for varying weights of a Bergsonian welfare functional in order to get the Pareto efficient frontier. (see: Kreps (1990), pp. 169-174; the classical reference is Wilson (1968).
proportional to its own risk tolerance divided by the overall risk tolerance of society, if the parties’ preferences over lotteries exhibit constant risk aversion (and therefore can be represented by an exponential utility function). In the model presented here, the exponential utility function was assumed right from the beginning. Moreover, a normally distributed output was assumed. The advantage of this set of assumptions - as was proven in the appendix to the previous Section - is that certainty equivalents can be calculated as a simple function of mean and variance. Consistent with the above-cited fact of syndicate theory, distributional assumptions (in this case $\sigma^2$) proved to be irrelevant for the optimal sharing rule.

However, this changes as soon as the possibility of portfolio effects is allowed for. In this case, the modeling framework chosen is extremely convenient as it readily admits the use of portfolio theory from finance which is cast in a mean-variance framework.

**2.4 The Optimal Contract**

**2.4.1 Introduction**

In the above model an important restriction was imposed. Only linear incentive schemes were considered. This assumption is arbitrary, unless it is possible to show that linearity is a feature of optimal contracts. Unfortunately, quite to the contrary, the following propositions can be derived:

7. If effort is **contractible**, the optimal compensation is a flat fee. If effort is **uncontractible**, optimal compensation varies with outcome.

8. If effort is **uncontractible**, optimal compensation depends on outcome through the likelihood ratio. If interpreted in terms of an **inference process**, any outcome that makes the principal revise upwards his beliefs with respect to the probability of high effort will be rewarded.

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311 This cannot be taken for granted, because $\mu, \sigma$ - analysis is not generally equivalent to the concept of risk averseness (see Rothschild, Stiglitz (1970)). Another example besides the normal-exponential setting is the quadratic utility function which admits a $\mu, \sigma$ - representation for any distributional assumption but has otherwise awkward properties such as increasing absolute risk averseness (see: Schneeweß (1967), pp. 95ff.; Feldstein (1968). Better suited is a power function combined with a lognormal probability distribution (see: Schneeweß (1967), pp. 145 ff).
9. The relationship between the optimal sharing rule and outcome is working through the **information content of outcome** which depends on distributional assumptions. Physical properties of outcome (like quantity) are not interesting as such but only to the extent that they carry information. Therefore, the sharing rule which links compensation to outcome is very contrived and sensitive to distributional assumptions.

10. In conclusion of propositions 7-9, it can be said that few general constraints on the shape of optimal incentive schemes can be derived. Utility functions and distributions must be specified in order to arrive at meaningful constraints. What is more, optimal contracts are very sensitive to these assumptions.

11. No natural settings seem to exist for which linearity is optimal. **Linear incentive schemes** can, however, be argued to be relatively robust to changing distributional assumptions and to assumptions concerning the richness of the action space. Transaction cost arguments also favor simple incentive schemes.

12. Non-distributional assumptions like the richness of the action space affect optimal contracts.

13. For \( y \) to be **valuable information**, it must affect posterior assessment. It must be a signal for effort choice without existing information being a sufficient statistic for \( y \) with respect to \( H \) and \( L \).

The proof for these propositions can only be sketched\(^ {312} \).

### 2.4.2 Mechanics of the Optimal Sharing Rule

The first step will be to set up a control theoretic model in order to derive properties of the optimal sharing rule. This will lead to a **statistical interpretation** of the basic agency model\(^ {313} \). This interpretation is key to the understanding of the mechanics of incentive schemes relevant to the following sections, which is why it will be treated at some length. This interpretation gives an intuitive explanation of the above proposition that only very few general

\(^{312}\) An excellent review of the following discussion can be found in Hart, Holmström (pp. 79-97, 1987).

\(^{313}\) The statistical interpretation follows the treatment of Hart, Holmström(1987)
constraints can be derived as for the shape of the optimal contract, and none of them very meaningful.

The model will be slightly different from the model used above. It is assumed that the agent cannot choose from a continuum of actions but only between two effort levels. Either he can be hard-working or lazy and this time the agent, by choosing his level of effort, is effectively choosing two probability distributions over a continuous support set of signals. This allows studying more closely the effects of distributional assumptions by allowing a much higher variety of distributions.

On a technical level, this leads to the following changes compared to above: As there is a finite action space, the number of incentive constraints will be finite (in this case there is only one incentive constraint) and therefore no first-order approach is needed. The continuity of the set of signals makes it impossible to stipulate that, for any choice of effort, any output arises with positive probability because probabilities then would not add up to zero. Instead, it is assumed that the support sets of signals are the same and that both density functions are strictly positive\textsuperscript{314}.

As above, the principal has to solve the following maximization problem\textsuperscript{315}:

\[
\max \int (x-s(x)) f_H(x) \, dx \\
\text{s.t. } IC : \int u(s(x))\left[f_H(x) - f_L(x)\right] \, dx - c_H + c_L \geq 0 \\
PC : \int u(s(x))f_H(x) - c_H - u_0 \geq 0
\] (5.1)

where \( x \) is the signal (here simply output), \( s(x) \) is the sharing rule, \( f_H(x) \), \( f_L(x) \) are the density functions over output for effort levels \( H \) and \( L \) respectively. \( f_H \) strictly dominates \( f_L \) in the sense of stochastic dominance (\( F_H(x) > F_L(x) \)). \( c_H, c_L \) is the disutility of effort for effort levels \( H \) and \( L \) respectively as measured on the scale of disutility. Both the principal and the agent are v-N-M expected utility maximizers. The principal is assumed to be risk-neutral. The agent is risk-averse. His utility function \( u(\cdot) \) is increasing, twice continuously differentiable and concave (\( u'(\cdot) > 0, u''(\cdot) < 0 \)). \( u_0 \) is the agent’s reservation utility.

\textsuperscript{314} see Kreps (1990), p. 604 (he is more formal by stipulating that the “likelihood ratios are uniformly bounded and bounded away from zero)

\textsuperscript{315} This is the parameterized distribution formulation of the agency problem in a more general version than above, see Holmström (1979). Tirole (2000) also used this formulation.
As above, it can be argued that both constraints must be binding. The incentive constraint must bind because increasing compensation differentials above the level needed in order to induce the agent to implement the high effort would unnecessarily increase the risk exposure of the agent and therefore expected payment by the principal. Also, the participation constraint must hold with equality, because otherwise the principal’s expected payment could be reduced by lowering compensation at every signal by the same amount without affecting incentives.

Letting $\mu$ and $\lambda$ be the Lagrangean multipliers\(^{316}\) for constraint (5.2) and (5.3) respectively, it can be written:

$$L(s(x), \mu, \lambda) = \int (x-s(x))f_H(x) + \mu u(s(x))[f_H(x) - f_L(x)] + \lambda u(s(x))f_H(x) + \mu(c_L - c_H) - \lambda(c_H + u_0)dx$$

As $\partial L/\partial s(x) = 0$, it can be written:

$$-f_H(x) + \mu[f_H(x) - f_L(x)]u'(s(x)) + \lambda f_H(x)u'(s(x)) = 0 \hspace{1cm} (5.5)$$

Dividing both sides by $f_H(x)u'(s(x))$ and rearranging yields the following condition for optimal sharing rules:

$$\frac{1}{u'(s(x))} = \lambda + \mu \left(1 - \frac{f_L(x)}{f_H(x)}\right) \hspace{1cm} (5.5)'$$

If effort is contractible, there will be no incentive problems (as the principal will use a forcing contract). Therefore, the incentive constraint will not bind. From the complementary slackness conditions it can be followed that this implies $\mu = 0$. If $\mu = 0$, the optimal compensation $s(x)$ will be a flat fee.

If effort is not contractible, there will be an incentive problem. The incentive constraint will bind. If $\mu = 0$ the agent will choose $L$ in violation of the incentive constraint. Therefore $\mu > 0$. But then, optimal compensation $s(x)$ will vary with outcome $x$.

\(^{316}\) If constraints cannot be argued to hold with equality, Kuhn-Tucker conditions have to be used.
Proposition 7: If effort is contractible, the optimal compensation is a flat fee. If effort is uncontractible, optimal compensation varies with outcome.

This is the same result as in (2.1), only this time the expression also gives insight into the “mechanics” of the relationship. The sharing rule $s(x)$ depends on $x$ through:

$$\frac{f_L(x)}{f_H(x)}.$$

This expression is familiar from statistical inference and is called the likelihood ratio. "It reflects how strongly $x$ signals that the true distribution from which $x$ was drawn is $f_L$ rather than $f_H$. A high likelihood ratio speaks for $L$ and a low for $H."^{317} It can be seen that, in an optimal incentive scheme, the agent is punished for a high likelihood ratio and rewarded for a low likelihood ratio:

$$\frac{f_L(x)}{f_H(x)} \Downarrow \Rightarrow \frac{1}{u'(s(x))} \Uparrow \Rightarrow u'(s(x)) \Downarrow \Rightarrow s(x) \Uparrow, \text{ for } u''(\cdot) < 0.$$

Thus, any outcome that suggests that high effort was most likely chosen is rewarded, which is a quite intuitive result.

It is possible to rewrite (5.5) in a way which stresses even more the analogy to statistical inference. Letting $\gamma'$ be the prior of $H$ and $\gamma'(x)$ the posterior of $H$ if $x$ was observed and applying Bayes' rule gives (see Exhibit 1):

$$\gamma'(x) = \frac{\text{Prob}(H | x)}{\text{Prob}(H) \text{Prob}(x | H) + \text{Prob}(L) \text{Prob}_L(x | L)} = \frac{\gamma f_H(x)}{\gamma f_H(x) + (1-\gamma) f_L(x)}.$$  (5.6)

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\(^{317}\) Hart/ Holmström (1987), p. 80
Prob(H) = \gamma

Prob(L) = 1 - \gamma

Exhibit 1: Agent chooses a Distribution of Outcomes

Rearranging yields:

\frac{f_L(x)}{f_H(x)} = \left( \frac{1}{\gamma'(x)} - 1 \right) \frac{\gamma}{1 - \gamma}.

(5.6')

Inserting (5.6)' into (5.5)' yields:

\frac{1}{u'(s(x))} = \lambda + \mu \left[ 1 - \left( \frac{1}{\gamma'(x)} - 1 \right) \frac{\gamma}{1 - \gamma} \right].

(5.7)

Rearranging yields:

\frac{1}{u'(s(x))} = \lambda + \mu \left[ \frac{\gamma'(x) - \gamma}{\gamma'(x)(1 - \gamma)} \right].

(5.7)'

It becomes clear that compensation rises if the output makes the principal revise upwards his beliefs with respect to the probability of high effort.

Proposition 8: If effort is uncontractible, optimal compensation depends on outcome through the likelihood ratio. If interpreted as an inference process, any outcome that makes the principal revise upwards his beliefs with respect to the probability of high effort will be rewarded.

It must be stressed that the principal does not in fact infer. He can perfectly predict the actions of the agent, but the optimal incentive scheme is designed as if
it was a reaction to the inferences of the principal. This interpretation is instructive to understand why there are only few general constraints\textsuperscript{318} and none of them meaningful: The relationship between the optimal sharing rule and outcome is working through the information content of outcome. In other words: The physical properties of the outcome (e.g. monetary success of the project) are only interesting to the extent that they carry information about which action was chosen by the agent; but this information content depends exclusively on distributional assumptions. It is not even possible to derive that compensation is always increasing in output. Consider e.g. distribution $f_H$, $f_L$ where $f_H(x) = f_L(x + 1)$. There is stochastic dominance ($F_H > F_L$) but if it is e.g. assumed that $f_H$, $f_L$ are bimodal then it becomes clear that there is no monotonicity of $s(x)$ in $x$ (see Exhibit 2). Of course, it is possible to make the assumption that the likelihood ratio is monotone in outcome\textsuperscript{319} to ensure monotonicity of $s(x)$\textsuperscript{320}.

\begin{center}
\textbf{Exhibit 2: No Monotonicity of Compensation in Outcome despite Stochastic Dominance}\textsuperscript{321}
\end{center}

\textbf{Proposition 9: The relationship between the optimal sharing rule and outcome is working through the information content of outcome which depends on distributional assumptions. Physical properties of outcome (like quantity) are not interesting as such but only to the extent that they carry information.}

\textsuperscript{318} see Grossman, Hart (1983)

\textsuperscript{319} This is called the Monotone Likelihood Ratio Property (MLRP), which is attributed by some to Milgrom (1981) and by others to unpublished work of Mirrlees.

\textsuperscript{320} In fact, it was already mentioned above that in the case of more than two available actions another assumption called the "concavity of the distribution function assumption" (see also footnote 266) is needed to ensure monotonicity (see Kreps (1990), pp. 596-598; Grossman, Hart (1983), example 1).

\textsuperscript{321} For a very intuitive display of this property see Mas-Colell, Whinston, Green (1995), p. 486.
Therefore, the sharing rule which links compensation to outcome is very contrived and sensitive to distributional assumptions.

Therefore, in order to derive meaningful constraints, additional distributional assumptions have to be made. Yet, it becomes clear from the above that derived optimal incentive schemes are very sensitive to these distributional assumptions.

Proposition 10: In conclusion of propositions 7-9, it can be said that little general constraints on the shape of optimal incentive schemes can be derived. Utility functions and distributions must be specified in order to arrive at meaningful constraints. Optimal contracts, however, are very sensitive to these assumptions.

2.4.3 The Case for Linear Contracts

The sensitivity of optimal incentive schemes to distributional assumptions suggests a great variety of different incentive schemes\(^{322}\). This contrasts to the real world experience where only a few relatively simple incentive schemes like linear incentive schemes, step-functions and flat fee contracts can be found.

One obvious reason for this conflicting evidence can be transaction cost arguments. Optimal contracts can sometimes be rather complex. People tend not to conclude such contracts.

Yet, there are more subtle explanations. One can be the robustness of the incentive scheme: Some schemes may be more robust to changing assumptions with respect to probability distributions and utility functions\(^{323}\) than others. If these distributional assumptions require more accurate information than is practically available, there is an advantage to using schemes which are quite good for a whole range of assumptions. It can e.g. be shown that, for a linear production function and a normally distributed error term (a quite natural formulation), linear incentive schemes are always dominated by step function schemes which can approximate first best arbitrarily closely\(^{324}\). This is because, at very low outcomes, the likelihood ratio decreases to a point where one can almost act as if non-compliance was determined with zero chance of error. Harsh penalties can be

\(^{322}\) see Hart, Holmström (1987), p.91 n whose line of argument is followed here

\(^{323}\) see e.g. Kreps(1990), p. 612

\(^{324}\) Mirrlees (1974)
inflicted for these outcomes, while the payment is a flat fee over all other outcomes. Therefore, perfect risk sharing can almost be achieved and the agent will choose high effort because low effort will increase the probability of harsh punishment. There are several reasons why this example is not realistic. It may not be possible to impose harsh enough penalties because of bankruptcy constraints, or maybe the likelihood ratio is bounded, but this example highlights a more fundamental problem: Step function schemes are an extreme case of fine tuning. While they are better than linear schemes in all cases if the range of outcomes for inflicting the penalty and the amount of the penalty are fixed at the optimal level, it is worse than linear schemes in most cases, where the assumptions are only rough estimates as is almost always the case in the real world.

Another robustness argument refers to the richness of the action space. It is often observed in economics that measures do not work where the agents have enough options to circumvent them. To capture the intuition it is extremely difficult for a government to tax the extremely wealthy as they can always move their residence to another country. It is also impossible for a company to engage in price discrimination if reselling is permitted. In this case, any price discrimination is arbitrated away. Following the same rationale, it can be shown that step functions create path-dependent incentives. If it is assumed that an agent can observe progress while the project is underway, he will quite often decide that he does not need to exert effort in the step function case. This is because in many situations he will either conclude that he will not be able to reach the threshold where he is paid or that he has already sufficiently surpassed it. So, he will neither do his best to prevent a bad situation from turning worse nor try to make a good situation even better. This path dependency of incentives does not apply for linear incentive schemes which keep incentives fairly constant. This result “illustrates a more general idea namely, that complicating the nature of the incentive problem can actually lead to simpler forms for optimal contracts”.

**Proposition 11:** No natural settings seem to exist where linear schemes are optimal. Linear incentive schemes, however, can be argued to be relatively robust to changing distributional assumptions and to assumptions concerning the richness of the action space. Transaction cost arguments also favour simple incentive schemes.

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325 Note that this is actually a multi-period model. For more on dynamic extensions see Chapter IV5.

It is questionable whether this very general conclusion in rescue of linear incentive schemes is justified given the very restrictive assumptions\textsuperscript{327}. It can be argued that the main contribution of this argument was to stress the importance of non-distributional assumptions such as the richness of the action space, when deriving optimal incentive schemes. To illustrate the idea, when designing incentives one is actually preparing a path which is meant to channel the behaviour of the agent. Every time he takes the decision, there have to be walls erected to ensure that he stays on the path, otherwise the agent gains control of the game and can manipulate outcome. This, however, poses the problem of predicting all the relevant options available to the agent. If one fails to do so, the scheme may break down.

\emph{Proposition 12: Non-distributional assumptions, like the richness of the action space, affect optimal incentive schemes.}

2.4.4 Valuable Information

A precise result can be derived about which parameters should enter into an optimal contract in the first place. Suppose \( y \) is another signal besides output \( x \) and that the joint density function is \( f_i(x, y) \) for \( i = L, H \). Then, analogous to (5.5)', it can be written:

\[
\frac{1}{u'(s(x))} = \lambda + \mu \left( 1 - \frac{f_L(x, y)}{f_H(x, y)} \right)
\]

(5.8)

As \( f_i(x, y) = f_i(x)f_i(y) \) the likelihood ratio can be written:

\[
\frac{f_L(x)f_L(y)}{f_H(x)f_H(y)}
\]

(5.9)

If \( f_H(y) = f_L(y) \), then they cancel out and the optimal contract should not depend on \( y \). This is an intuitive result: The condition means that \( y \) is just unrelated noise with respect to \( H \) and \( L \). It is obvious that such a variable would just add noise, increasing cost for the principal who has to compensate the agent for his risk exposure, without carrying any information.

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\textsuperscript{327} Gibbons (2001)
But it was already assumed above that $y$ is a signal. Therefore, $f_H(y) \neq f_L(y)$. Yet, it will be shown that being a signal is just a necessary condition for a contract parameter to provide valuable information.

As the condone probability of $y$ when $x$ was observed is:

$$f_i(y|x) = \frac{f_i(x,y)}{f_i(x)}$$  \hspace{1cm} (5.10)

Rearranging gives:

$$f_i(x,y) = f_i(y|x)f_i(x)$$  \hspace{1cm} (5.10)'

If $f_H(y|x) = f_L(y|x)$ it can be seen from (5.8) that the optimal contract does not depend on $y$. This condition says that $y$ is perfectly correlated with $x$ with respect to $H$ and $L$. All the information about effort choice that is contained in $y$ is already contained in $x$. So, $y$ offers no additional information. It is also said that $x$ is a sufficient statistic for $y$ with respect to $H$ and $L$. This leads to the following proposition.

**Proposition 13.: For $y$ to be valuable information it must affect posterior assessment. It must both be a signal for effort choice and the information already available may not be a sufficient statistic for $y$ with respect to $H$ and $L$.**

This result is also called the sufficient statistic result or the Holmström informativeness condition$^{328}$.

How can this result be interpreted? When designing the incentive scheme of a consultant who makes a cost cutting project there is no use in monitoring both the cost saved and the number of workers laid off. There is a deterministic relationship between the two variables. They are perfectly or very closely correlated, but it may make sense to monitor the hours worked by the consultant, formal consistency of the report, the extent and depth of analysis, the amount of relevant empirical material used, the numbers of interviews concluded and the satisfaction of the people involved.

This information is valuable as it increases the ability to separate between high and low effort. It is as if inferences become more precise. Expression (5.8)

---

predicts that as the likelihood ratio decreases the optimal compensation differential is very high. This is because, as inferences are very precise, the chance of error is low and therefore the welfare loss due to imperfect risk-sharing can be kept low in spite of the wage differential. The principal can severely punish the agent if low outcome is observed.

Clearly there is also a cost in obtaining information and often the most valuable information is also the most expensive to obtain. So, directly observing effort would be the most valuable information, but it was assumed that it is uncontractible (read: can only be made contractible at prohibitive cost). If they came at the same cost, one would always prefer to do input monitoring to output monitoring. So, this result on its own only helps to determine the value of information but, in order to decide which parameter should enter the contract, one also has to consider the cost of contracting on it.

The result highlights another important point: It is not only sufficient to look if information is a signal. It must also be analysed whether the signal is not disturbed by the same noise. Beyond the obvious case cited above, this suggests that there is a decreasing marginal utility of information as with a reasonable number of variables considered, probably most of the uncertainty that can be filtered out has been filtered out. The method when considering adding a signal therefore is to ask what the risks are that disturb the relationship between the choice of effort and the signal, and if these risks are of different type and origin than in the relationship between effort and the existing signals. If this is the case, the signal is likely to be valuable.

2.4.5 Discussion

In this Section, a control theory model was set up to determine the optimal sharing rule. It was possible to reprove the results of earlier sections. If effort is contractible, flat fee contracts will be used. If it is not contractible it will depend on output, but contrary to earlier results it was also possible to shed some light on the mechanics of the optimal sharing rule. Compensation depends on output through the likelihood ratio. There are two implications: The optimal sharing rule can be understood quite intuitively as rewarding the agent if the signal makes it likely that high effort was chosen. But it also explains why little general constraints can be derived for the shape of the optimal sharing rule: It is very sensitive to specifications of utility functions and distributional assumptions. Although the optimal contract can only be derived to be linear under awkwardly
improbable assumptions\textsuperscript{329}, there are a number of reasons explaining the practical prominence of such contracts. First, there is the transaction cost argument. Setting up complex contracts is just too expensive. Second, linear contracts are argued to be relatively robust for a large number of settings. So, paradoxically, there are reasons to believe that adding complexity makes contracts simpler. But also non-distributional assumptions, like the number of different options available to the agent, affects the optimal incentive scheme. It was also shown that information is only valuable if it affects posterior assessment of the effort level chosen. Therefore, it must be related to effort choice, but it must also be impossible to perfectly infer the information - to the extent that it is relevant to this assessment - from information on variables already included into the contract. The same results can also be derived for the more general case of a continuous action space and a continuous set of outcomes\textsuperscript{330}, but this complication offers no additional economic insights\textsuperscript{331}.

### 2.5 Limitations and Extensions

A common assumption is that there is a \textit{comparative cost advantage} of output monitoring compared to input monitoring. Why should this be the case? In order to implement input monitoring, the principal has to watch the agent while performing the required task. This causes \textit{opportunity costs} to the principal. Still worse, if the principal does not know the production function of the agent, he may well watch the agent while performing a task but will be \textit{unable to interpret his actions} as to whether they are instrumental to achieve the required output. These costs are amplified by the fact that, usually, the very motivation to hire an agent in the first place was that the principal either did not want or could not perform the task himself. So, either the principal has something else to do, which means that his opportunity costs are high, or the performance of the task requires specialized knowledge that the principal does not possess. The latter case does make it difficult for the principal to monitor the agent effectively. Alternatively, the principal could hire \textit{other qualified agents} to do the monitoring for him, but then it may be difficult to prevent these monitoring agents from colluding with the operative agents. For all these reasons, input monitoring is likely to be very costly in many circumstances. On the other hand, output monitoring should be very easy. One has only to look to which extent the required result was achieved, provided that it can be properly defined. So, if a client hires a consultant to

\textsuperscript{329} see Hart, Holmström (1987) p. 81

\textsuperscript{330} see Kreps (1999). p. 608

\textsuperscript{331} see Hart, Holmström (1987), p. 83 who come to this conclusion.
perform a cost cutting project, it will be much easier for the client to evaluate how much cost was reduced than to interpret the wide variety of single measures the consultant takes to achieve his goal.

Having clearly established the intuition for comparative cost advantage of output monitoring compared to input monitoring in a wide variety of situations, it may come as a surprise that input monitoring can theoretically infinitely approximate first best. This is because the agent, in his decision on whether to cheat or not, will weigh the benefits of cheating (in the case of shirking reduced disutility of effort) against the expected value of punishment. Therefore, if harsh enough punishments are announced, the probability of detection and therefore the number and thoroughness of inspections can be infinitely reduced. This is the first line of attack against the model presented above on the grounds that input monitoring need not be more expensive than output monitoring. In this case, there seems no rationale for output monitoring. If it is not cheaper, it will only provide an additional drawback: imperfect risk sharing.

Yet, the second line of attack against the above model disputes just that. It was argued that input monitoring always establishes the truth, while output monitoring is prone to error. This is a crucial point, because it was shown above that the driving force behind imperfect risk sharing in output monitoring is the possibility of error in judgement and the subsequent punishment of the innocent. If it can be shown that output monitoring can achieve perfect accuracy, or that input monitoring is prone to error as well this distinction breaks down. In fact, both can be shown to be plausible assumptions in some circumstances: Output monitoring will be perfectly accurate in the case of deterministic production functions, but also if there is shifting support. Indeed, the above argument of Mirrlees on step-functions is in the same spirit. On the other hand, it seems implausible to assume that input monitoring will be able to prove cheating at 100%. There will always be judgement, inferences, circumstantial evidence. Whatever the process, there is a chance of error. Therefore, the above assumption will need to be relaxed to allow for error in input monitoring.

These arguments appear construed, and in fact they are. In general, input monitoring will be more costly and output monitoring more prone to error. Ignoring these arguments, therefore, seems to be a justified abstraction, but there is still some merit in taking them seriously. In a nutshell, they say that, regardless of whether one is looking at input monitoring or at output monitoring, there are two relevant issues: Error in judgement and cost of monitoring, and that under

332 see next Section
some circumstances both schemes fare equally well or badly on these two dimensions. By acknowledging that there generally is a distinction between input monitoring and output monitoring, one is actually saying that these circumstances will rarely be present. Understanding why this is the case helps to identify other relevant situational variables which influence the problem of optimal contracting.

Extensions to the classic agency model involve the role of the bankruptcy constraint, the role of error in the monitoring process which will be treated in Chapter (3), distortive effects of contracting (Chapter 4) and extensions beyond the one-shot relationship (Chapter 5).

3  Error in judgement, Bankruptcy

3.1  Input Monitoring

3.1.1  Introduction

In this Chapter it will first be argued that input monitoring achieves first best if harsh enough punishment is feasible and error in judgement can be excluded. Clearly, these assumptions are unrealistic. Bankruptcy and legal constraints set bounds to the extent of punishment. It will be shown that if a bankruptcy constraint is introduced, input monitoring will be costly. Alternatively, if error in judgement is allowed for, input monitoring will be costly even if there is no bankruptcy constraint. One should note the interesting twist in this argument: The first line of attack against costly input monitoring involves schemes with high punishment. It was argued before that, even if error in judgement cannot be excluded, it is possible to safely abstract from it because its probability is low. As soon, however, as schemes with very high punishments are introduced, even small chances of error will lead to substantial cost due to imperfect risk-sharing.

The following propositions will be derived:

14. Input monitoring can achieve first best if harsh-enough punishments are feasible and error can be excluded.

333 see e.g. Holmström, Milgrom (1991), Gibbons (2001)
334 see e.g. Bull (1987), Holmström (1999), Levin (2003), Lazear, Rosen (1981)
335 It is sometimes assumed in behavioural economics that probability of detection may not fall below a certain threshold in order to be effective, regardless of expected value, because otherwise it will not be taken seriously by the agent. Having mentioned it, this criticism will subsequently be ignored.
15. If a bankruptcy constraint is assumed, there is always a welfare loss as the optimal frequency of inspections does not tend to zero, resulting in direct costs of monitoring.

16. If there is a bankruptcy constraint and the cost of the monitoring technology is high enough, “efficiency wages” have to be paid in order to provide incentives. There are situations where no trade occurs, although there are potential gains of trade, resulting in welfare loss. This is because the principal’s profit would turn negative.

17. Monitoring cost rises in the scope of cheating (Δ), and decreases in the sum of punishment and compensation (d + s). This implies that compensation is a substitute for punishments, and “efficiency wages” are paid where the scope for punishment is limited due to bankruptcy and other legal constraints.

18. Monitoring cost is a decreasing and concave function in agent risk averseness. If it is possible to impose high punishment in situations where potential damage from cheating is high, the cost reduction effect is most powerful.

19. First best can also be infinitely approximated in the case where error is permitted if the agent is risk-neutral and there is no bankruptcy constraint.

20. In the case of a risk-averse agent, there will be a welfare loss if error in judgement is allowed for, even if there is no bankruptcy constraint.

3.1.2 Modeling Assumptions

It is assumed that if the agent shirks he can get a benefit of 1 in addition to his agreed-upon compensation s. In order to prevent this from happening, the principal installs monitoring technology. If it is established by the monitoring process that shirking occurred, the agent receives no compensation s and has to pay a fine d. The fine will almost always be positive. It will be allowed, however, that d is negative. This is the case if legal constraints exist that do not allow the agent to be punished but only make it possible to withhold part of the agreed-upon
salary \( s \)\(^{336}\). But \( d + s > 0 \), because never will the principal have to pay a bonus in excess of salary in the event of punishment.

By choosing the monitoring technology, the principal faces a **cost-quality trade-off**. The quality of a monitoring technology is high if there is a low probability of error. Error occurs if the agent is not punished although he did shirk or if he is punished although innocent. It is assumed that the principal faces a monitoring technology which establishes the truth at a probability of \( p(c) \) and errs at a probability of \( 1 - p(c) \), with \( c \geq 0 \) being the cost of investment in the monitoring technology\(^{337}\). It is assumed that the **probability of erring** is the same whether the agent is innocently punished or gets away with shirking\(^{338}\). Two cases will be considered. In the first case, the probability of erring will be zero. This is the traditional assumption of input monitoring.

\[
p = 1, \quad c = \bar{c}\quad (7.1)
\]

In the second case, the range of \( p(c) \) is the half-open interval **between 0.5 and 1**, with 1 as the upper boundary which can be infinitely approximated, but never reached. It is clear that the probability of establishing the truth cannot be less than 0.5. This is because, even if nothing is invested in the technology

\(^{336}\) It can be imagined that if it is determined in court that the agent did shirk, the judge, far from allowing the principal to punish the agent, actually only rules a fraction of the agreed-upon compensation can be withheld.

\(^{337}\) Later, a distinction is introduced between monitoring technology and monitoring scheme. A monitoring scheme is comprised of both monitoring technology and frequency of inspections.

\(^{338}\) **This assumption is not innocent** and needs to be discussed: If the probability of error is 20%, then a shirking agent will get away with it in 20% of the cases and an honest agent has a chance of 20% to be punished nevertheless. If the agent can challenge this decision in court, there may be a problem with monitoring technologies that seem to be very prone to error, especially in a **system obsessed with truth beyond doubt**. So, it could be that, in case a punishment is imposed, the court will only uphold it in 50% of all cases. Then, the probabilities of error would change to 60% and 10%. This raises the interesting problem of the coexistence of private settlement mechanisms and public courts. Parties might agree on a rather error prone monitoring scheme because it is cheap and find some sort of arrangement to compensate for the extra risk, but if the recourse to the courts cannot be excluded, the arrangement might break down. The problem will decrease if monitoring schemes' error comes closer to zero. Another problem arises if the principal controls the monitoring process. He will then have the incentive to always report cheating. Reputation concerns might prevent this, but in a one-shot relationship this clearly is a risk. So, it has to be assumed that the monitoring process is objective and transparent. If necessary, procedural rules can be enforced by the courts. In conclusion, the assumption seems rather strong, but it can be shown that the model does not depend on this in its outcome, so for convenience it is upheld.
(c = 0), it cannot be worse than tossing coins. p(·) is a positive, strictly increasing and concave function in c:

\[ p(c) \in [0, 5; 1); \lim_{c \to \infty} p(c) = 1; \quad p(0) = 0.5; \quad p'(\cdot) > 0; \quad p''(\cdot) < 0. \quad (7.2) \]

The principal has a possibility to save cost, other than keeping c low. He can randomize inspections, which means that he carries out inspections only at a fraction \( \alpha \) of cases. It is further assumed that all costs of inspection are variable. So the cost of the monitoring scheme will be \( \alpha c \).

It is further assumed that the principal’s and the agent’s preferences over lotteries satisfy the v-N-M axiom and that the principal is risk-neutral and the agent risk-averse. This is in line with traditional assumptions.

In Exhibit 3, the pay-offs for the agent and the principal are shown for the different cases. Outcome is written as the vector:

\[
\begin{pmatrix}
\text{pay-off agent} \\
\text{pay-off principal}
\end{pmatrix}
\]

One can now proceed to set up the maximization problem: The agent will only abstain from shirking if the incentive constraint is fulfilled, i.e. if the expected utility of not shirking is higher than the expected utility of shirking:

\[
\alpha \left[ 1 - p(c) \right] u(-d) + (1 - \alpha \left[ 1 - p(c) \right]) u(s) \geq
\left[ 1 - \alpha p(c) \right] u(s + \Delta) + \alpha p(c) u(-d)
\quad (7.3)
\]

Rearranging gives:

\[
\alpha (1 - 2p) u(-d) + u(s) - \alpha (1 - p) u(s) - (1 - \alpha p) u(s + \Delta) \geq 0 \quad (7.3)'\]

The relevant pay-off for the participation constraint and the principal’s objective function is the pay-off in the case of “no shirking”. This is because, otherwise, no incentive constraint would be needed and investment in monitoring would be zero. The agent will only accept the contract if the participation constraint is fulfilled:

\[
\alpha (1 - p) u(-d) + u(s) - \alpha (1 - p) u(s) - u_0 \geq 0 \quad (7.4)
\]
The principal’s objective function can be written as:

\[
\max \alpha (1-p)(x+d)+[1-\alpha (1-p)](x-s)-\alpha c
\]

(7.5)

Rearranging yields:

\[
\max \alpha (1-p)(d+s)+(x-s)-\alpha c
\]

(7.5')

Therefore, the **maximization problem** can be set up as:

\[
\max \alpha (1-p)(d+s)+(x-s)-\alpha c
\]

(7.6)

IC: \(\alpha (1-2p)u(-d)+u(s)-\alpha (1-p)u(s)-(1-\alpha p)u(s+\Delta) \geq 0\) (7.7)

PC: \(\alpha (1-p)u(-d)+u(s)-\alpha (1-p)u(s)-u_0 \geq 0\) (7.8)
3.1.3 Absence of both Error and Bankruptcy Constraint

In the first case that is considered, it is assumed that a perfect\textsuperscript{339} monitoring mechanism exists at a given cost:

\[ p(\bar{c}) = 1, \ c=\bar{c} \]  

(7.9)

It is further assumed that there is no bankruptcy constraint. So, any punishment is feasible. The principal’s problem therefore is to determine the optimal frequency of inspections \( \alpha \), the optimal salary \( s \) and the optimal punishment \( d \).

Inserting (7.9), the \textbf{maximization problem} can be stated as follows:

\[ \max_{\alpha,s,d} x - s - \alpha \bar{c} \]  

(7.10)

s.t. IC: \( -\alpha u(-d) + u(s) - (1-\alpha)u(s+\Delta) \geq 0 \)  

(7.11)

PC: \( u(s) - u_0 \geq 0 \)  

(7.12)

Letting \( \mu_1 \) and \( \mu_2 \) be the Lagrangean multipliers for (7.11) and (7.12), the following conditions must hold for an optimal contract:

\[ \frac{\partial L}{\partial \alpha} = -\bar{c} + \mu_1 \left( u(s+\Delta) - u(-d) \right) = 0 \]  

(7.13)

\[ \frac{\partial L}{\partial s} = -1 + \mu_1 u'(s) - \mu_1 (1-\alpha)u'(s+\Delta) - \mu_2 u'(s) = 0 \]  

(7.14)

\[ \frac{\partial L}{\partial d} = \mu_1 \alpha u'(-d) = 0 \]  

(7.15)

Note that the multipliers have to be non-negative:

\[ \mu_1 \geq 0, \ \mu_2 \geq 0. \]  

(7.16)

The complementary slackness conditions are given by:

\[ \mu_1 \left[ -\alpha u(-d) + u(s) - (1-\alpha)u(s+\Delta) \right] = 0 \]  

(7.17)

\[ \mu_2 [u(s) - u_0] = 0 \]  

(7.18)

\textsuperscript{339} i.e. zero error probability
Solving (7.13) for \( \mu_1 \) yields:

\[
\mu_1 = \frac{\bar{c}}{u(s + \Delta) - u(-d)} > 0 \tag{7.13}'
\]

which is positive as can easily be seen recalling that \( d + s > 0 \). This also means that the incentive constraint binds.

As \( u'(-d) \neq 0 \) it can be followed from (7.15) that either \( \alpha \) or \( \mu_1 \) must be 0. \( \mu_1 = 0 \) contradicts (7.13)' and \( \alpha = 0 \) cannot be true, because it is impossible to induce the agent not to shirk with zero probability of inspections. Therefore, the Lagrangean conditions cannot hold, which means that there is no optimum for this problem.

Even if there is no optimum, interesting implications can be derived: In order to induce the agent not to shirk, the incentive constraint must hold. Solving the incentive constraint (7.11) for \( \alpha \) gives:

\[
\alpha \geq \frac{u(s + \Delta) - u(s)}{u(s + \Delta) - u(-d)} \tag{7.11}'
\]

It is evident from the objective function (7.10) that \( \alpha \) will always be chosen at the lowest possible level. The smallest value for \( \alpha \) for which the incentive constraint holds is given by the following condition:

\[
\alpha = \frac{u(s + \Delta) - u(s)}{u(s + \Delta) - u(-d)} \tag{7.11}''
\]

Inserting (7.11)'' into (7.10) yields:

\[
x - s - \frac{u(s + \Delta) - u(s)}{u(s + \Delta) - u(-d)} \bar{c} \tag{7.10}'
\]

Assuming the agent to be risk-neutral, (7.10)' simplifies to:

\[
x - s - \frac{s + \Delta - s}{s + \Delta + d} \bar{c} = x - s - \frac{\Delta}{s + \Delta + d} \bar{c} \tag{7.10}''
\]

The optimal salary \( s^* \) for the principal to offer if he wants the incentive constraint to hold for the lowest possible \( \alpha \) must satisfy the first order condition:
\[
\frac{\partial \Pi}{\partial s} = -1 - \frac{-\Delta}{(s^* + \Delta + d)^2} c = 0
\]  

Rearranging gives:

\[
|s^* + \Delta + d| = \sqrt{c\Delta}
\]  

Two cases must be distinguished:

1. \(s^* + \Delta + d = -\sqrt{c\Delta}\)  
2. \(s^* + \Delta + d = \sqrt{c\Delta}\)

**Case (1) is impossible** for \(d > -s\) and \(\Delta > 0\). Solving (2) for \(s^*\) yields:

\[
s^* = \sqrt{c\Delta} - \Delta - d
\]

The contract must also satisfy the participation constraint. Solving the participation constraint for \(s\) yields:

\[
s \geq \bar{u}(u_0) = \bar{s}
\]

Salary \(s^*\) violates the participation constraint if:

\[
s^* < \bar{s}
\]

In this case, the wage is set to the reservation level, making the incentive constraint loose. In other words: Whenever the wage, necessary in order to meet the participation constraint, is higher than is needed to induce the agent not to shirk, the incentive constraint will also hold. Therefore, the chosen wage will be:

\[
s^* = \max \left[\sqrt{c\Delta} - \Delta - d, \bar{s}\right]
\]

Inserting into the objective function (7.10) gives:

\[
\Pi = \begin{cases} 
  x + \Delta + d - 2\sqrt{c\Delta} & \text{for } \bar{s} < \sqrt{c\Delta} - \Delta - d \\
  x - \bar{s} - \frac{\Delta}{\bar{s} + \Delta + d} c & \text{otherwise}
\end{cases}
\]
The optimal $d$ must satisfy the first order condition:

$$\frac{\partial \Pi}{\partial d} = \begin{cases} 1 & \text{for } s < \sqrt{c\Delta} - \Delta - d \\ \frac{\Delta}{(s + \Delta + d)^2} \frac{c}{c} & \text{otherwise} \end{cases} \quad (7.24)$$

It can be seen that $\frac{\partial \Pi}{\partial d} > 0$. $\Pi$ is strictly increasing in $d$. Therefore, there cannot be a maximum. This is the same result that was obtained with the Kuhn-Tucker conditions above.

As $d$ rises $\bar{s} < \sqrt{c\Delta} - \Delta - d$ will not hold. Even in the extreme case where $c \to \infty, \Delta \to \infty$ with $d \to \infty$ the condition will be $\bar{s} < -\Delta$. This never holds as $\bar{s} > 0$. Therefore, the other case applies. In other words: If the participation constraint holds, the incentive constraint will always be loose: In any case where the agent accepts the contract he will also refrain from shirking.

It was already established that no maximum exists. Yet, it can be shown that there is an upper boundary:

$$\lim_{d \to \infty} \left( x - \bar{s} - \frac{\Delta}{\bar{s} + \Delta + d} \frac{c}{c} \right) = x - \bar{s} \quad (7.25)$$

Therefore, it is possible to infinitely approximate first best by infinitely increasing punishment in the case where there is no error in judgement and no bankruptcy or other legal constraints exist.

**Proposition 14**: Input monitoring can infinitely approximate first best if harsh enough punishments are feasible (no bankruptcy constraint) and error can be excluded.

### 3.1.4 Bankruptcy constraint

In this subsection, it is still assumed that a perfect monitoring mechanism is available at a given cost $c$. However, now a bankruptcy constraint is introduced. This can have many reasons. The capacity of individuals to absorb losses is limited either by nature or by law (limited liability). Legal provisions do not allow certain kinds of punishment. Therefore, the extent of punishment that can be imposed is limited ($d \geq \bar{d}$).
The result (7.24) of Section (3.1.3) still holds. \( \Pi \) is strictly increasing in \( d \), but \( d \) is \textbf{bounded} to reflect the bankruptcy constraint, \( d \in (-s, d_0] \). A maximum now exists as a \textbf{boundary solution}, \( d = d_0 \). The maximum amount of punishment will be imposed. But this time, as \( d \) cannot rise indefinitely it becomes clear from (7.25) that the frequency of inspections does not approach zero:

\[
\alpha = \frac{\Delta}{s + \Delta + d} \geq 0 \tag{7.26}
\]

Thus, it can be concluded that:

**Proposition 15:** If a bankruptcy constraint is assumed, there is always a welfare loss as the optimal frequency of inspections does not tend to zero, resulting in direct costs of monitoring.

But this is not the only source of welfare loss. In the above argument it could be shown that the incentive constraint was always loose if the participation constraint was met. If there is a bankruptcy constraint it can be seen that:

\[
\bar{s} \geq \sqrt{c \Delta - \Delta - d} \tag{7.27}
\]

will not always hold if for a given \( \bar{s} \) and \( \bar{d} \) the cost of the monitoring technology \( \bar{c} \) is high enough. More specifically, the pay level needed to provide incentives not to shirk will exceed the reservation level ("efficiency wages") if situational parameters are such that:

\[
\bar{c} > \frac{(\bar{s} + \Delta + \bar{d})^2}{\Delta} \tag{7.28}
\]

If, however, the \textbf{cost of investment in the monitoring technology is too high}, the contract will not be offered by the principal at all, as his optimal pay-off turns negative. To prevent this from happening, the following condition must hold:

\[
x + \Delta + \bar{d} - 2\sqrt{c \Delta} \geq 0. \tag{7.29}
\]
From \( d > -s \) and \( s < x \) follows \( d > -(x + \Delta) \) and therefore \((x + \Delta + d) > 0\). Thus (7.29) can be rearranged to yield:

\[
\bar{c} \leq \frac{(x + \Delta + \bar{d})^2}{4\Delta}.
\] (7.29)′

If (7.28) holds as was assumed, (7.29)′ only holds if:

\[
(x + \Delta + \bar{d})^2 > 4\left(\frac{s + \Delta + \bar{d}}{s + \Delta + \bar{d}}\right)^2.
\] (7.30)

It can be seen that there are situations with potential gains of trade \((x > \bar{s})\) where this condition does not hold.

**Proposition 16:** If there is a bankruptcy constraint and the costs of the monitoring technology are high enough, "efficiency wages" have to be paid in order to provide incentives. There are situations where no trade occurs, although there are potential gains of trade because the principal’s profit would turn negative, resulting in a welfare loss.

### 3.1.5 Extension: The role of Agent Risk Averseness

In order to study the role of risk averseness, the utility function will be specified. It is assumed that the agent's utility function is exponential:

\[
u(x) = 1 - e^{-rx}.
\] (7.31)

Inserting (7.31) into the expression for \( \alpha \) (see (7.11)′′) and rearranging yields (see Exhibit 4)

\[
\alpha = \frac{1 - e^{-r\Delta}}{e^{r(d+s)} - e^{-r\Delta}}.
\] (7.32)

The sensitivity of monitoring cost \( \alpha \) to changes of risk averseness \( r \) is given by the first derivative (see Exhibit 5):

---

340 It was already argued that otherwise punishment would lose its meaning.

341 Agreed-upon salary will never be higher than the gross outcome.
\[ \frac{\partial \alpha}{\partial \tau} = e^{(d+s)} \left[ (\Delta + d + s) e^{-\tau \Delta} - (d + s) \right] - \Delta e^{-\tau \Delta}. \] (7.33)

The expressions are a bit unwieldy. As the functions were specified, it is convenient to depict the functions to get an intuition of the relationships: It can be seen in Exhibit 4 that monitoring cost rises with the scope of cheating (\( \Delta \)), and decreases in the sum of punishment and compensation (\( d + s \)).

Exhibit 4: Monitoring Cost

This is a quite intuitive result. Perhaps the most surprising is that monitoring cost only depends on the sum of \( d \) and \( s \). This is, however, consistent with the result that efficiency wages are paid where the scope for punishment is limited due to bankruptcy and other legal constraints.

**Proposition 17:** Monitoring cost rises in the scope of cheating (\( \Delta \)), and decreases in the sum of punishment and compensation (\( d + s \)). This implies that compensation is a substitute for punishment and "efficiency wages" are paid where the scope for punishment is limited due to bankruptcy and other legal constraints.
The responsiveness of monitoring cost to changing risk averseness is depicted in *Exhibit 5*. It can be seen that the cost of implementing input monitoring decreases as the risk averseness of the agent increases:

\[
\frac{\partial \alpha}{\partial r} < 0 \forall d + s, \forall \Delta
\]  

(7.34)

Exhibit 5: Responsiveness of monitoring cost to changing risk averseness.

If it is possible to impose high punishment in situations where potential damage from cheating is high, this effect is most powerful.

In *Exhibit 6* the level of responsiveness of monitoring cost to changes in risk attitudes is shown as a function of risk averseness.

It becomes clear that the higher the level of risk averseness to begin with, the higher the decrease of monitoring cost from a small increase of risk averseness will be. Monitoring cost is a decreasing and concave function in agent risk averseness.
Exhibit 6: Responsiveness of Monitoring Costs to Changing Risk Averseness

Proposition 18: Monitoring cost is a decreasing and concave function in agent risk averseness. If it is possible to impose high punishment in situations where potential damage from cheating is high, the cost reduction effect is most powerful.

3.1.6 Presence of Error

If the possibility of error is allowed for, the maximization problem is a bit more complex:

\[
\begin{align*}
\max_{a,c,s,d} & \quad \alpha (1-p)(d+s) + x - s - \alpha c \\
\text{s.t.} & \quad \alpha (1-2p)u(-d) + u(s) - \alpha (1-p)u(s) - (1-\alpha p)u(s+\Delta) \geq 0 \\
& \quad \alpha (1-p)u(-d) + u(s) - \alpha (1-p)u(s) - u_0 \geq 0
\end{align*}
\]

Letting \( \mu_1 \) and \( \mu_2 \) be the Lagrangean multipliers for constraints (7.36) and (7.37), the following conditions must hold:
\[
\frac{\partial L}{\partial \alpha} = (1-p)(d+s) - c + \mu_1 \left[ (1-2p)u(-d) - (1-p)u(s) + pu(s+\Delta) \right] + \mu_2 \left[ (1-p)u(-d) - (1-p)u(s) \right] = 0 \\
\frac{\partial L}{\partial c} = -\alpha p'(d+s) - \alpha + \mu_1 \left[ -2\alpha p'u(-d) + \alpha p'u(s) + \alpha p'u(s+\Delta) \right] + \mu_2 \left[ -\alpha p'u(-d) + \alpha p'u(s) \right] = 0 \\
\frac{\partial L}{\partial s} = \alpha(1-p) - 1 + \mu_1 \left[ u'(s) - \alpha(1-p)u'(s) - (1-\alpha p)u'(s+\Delta) \right] + \mu_2 \left[ u'(s) - \alpha(1-p)u'(s) \right] = 0 \\
\frac{\partial L}{\partial d} = \alpha(1-p) + \mu_1 \left[ \alpha(1-2p)u'(-d)(-1) \right] + \mu_2 \left[ \alpha(1-p)u'(-d)(-1) \right] = 0
\]

Rearranging (7.38) and (7.39), it can be written:

\[
d+s - \frac{c}{1-p} + \mu_1 \left[ \frac{1-2p}{1-p}u(-d) - u(s) + \frac{p}{1-p}u(s+\Delta) \right] + \mu_2 \left[ u(-d) - u(s) \right] \\
- (d+s) - \frac{1}{p'} + \mu_1 \left[ -2u(-d) + u(s) + u(s+\Delta) \right] + \mu_2 \left[ -u(-d) + u(s) \right]
\]

Adding (7.38)' and (7.39)' and solving for \( \mu_1 \) yields:

\[
\mu_1 = \frac{p'c - p + 1}{p'[u(s+\Delta) - u(-d)]}
\]

Rearranging (7.41) and solving for \( \mu_2 \) gives:

\[
\mu_2 = \frac{1}{u'(-d)} - \mu_1 \left( 1 - \frac{p}{1-p} \right)
\]

It was assumed that the chance of error in judgement \( 1-p \) is a function of the investment in the monitoring technology \( c \). If nothing is invested, the chance of error is 50% (like tossing coins). As the investment is increased, the chance of error decreases, but will never be zero. Therefore, \( p(c) \) must be a function which takes the value 0.5 for \( c = 0 \) and asymptotically approaches 1 as \( c \to \infty \).
In order to facilitate the argument \( p(c) \) is specified by a simple function, fulfilling these properties:

\[
p(c) = 1 - \frac{0.5}{1 + c} = \frac{2c + 1}{2(1 + c)}
\]

(7.43)

\[
p'(c) = \frac{1}{2(1 + c)^2}
\]

Inserting (7.43) in (7.42) and (7.41)' gives:

\[
\mu_1 = \frac{2c + 1}{u(s + \Delta) - u(-d)}
\]

(7.44)

\[
\mu_2 = \frac{1}{u'(-d)} + 2\mu_1c
\]

(7.45)

Inserting (7.44) into (7.45) gives:

\[
\mu_2 = \frac{1}{u'(-d)} + \frac{4c^2 + 2c}{u(s + \Delta) - u(-d)}
\]

(7.45)'

As \( c > 0, d > -s \) and \( u'(\cdot) > 0 \) it can be followed that both \( \mu_1 \) and \( \mu_2 \) are strictly positive. From the complementary slackness conditions, it follows that both constraints bind.

At this point a methodological remark is warranted: Microeconomic analysis attempts to isolate effects. The effect of a bankruptcy constraint was studied above. A perfect monitoring mechanism was assumed in order to make sure that only the effect produced by the bankruptcy constraint is considered. Now, the focus of analysis is the effect of an imperfect monitoring mechanism which allows for error in judgement. In order to isolate this effect it is assumed that no bankruptcy constraint exists. Conditions (7.44) and (7.45)' will therefore be analysed assuming infinite punishment potential \( (d \to \infty) \).

If the agent is risk neutral and because \( u(x) = x \), \( u'(x) = 1 \ \forall x \), \( \mu_1 \) and \( \mu_2 \) will simplify to:

\[
\mu_1 = \frac{2c + 1}{s + \Delta + d}
\]

(7.46)

\[
\mu_2 = 1 + \frac{4c^2 + 2c}{s + \Delta + d}
\]

(7.47)
It can easily be seen that for \( d \to \infty \), \( \mu_1 \to 0 \) and \( \mu_2 \to 1 \).

Now the case is considered, where the agent is risk averse:

\[
\mu_1 = \frac{2c + 1}{u(s + \Delta) - u(-d)} \quad \text{(7.44)}
\]

\[
\mu_2 = \frac{1}{u'(-d)} + \frac{4c^2 + 2c}{u(s + \Delta) - u(-d)} \quad \text{(7.45)'}
\]

As \( u(-d) \to -\infty \) and \( u'(-d) \to \infty \) for \( d \to \infty \), it can easily be seen that \( \mu_1 \to 0 \) and \( \mu_2 \to 0 \).

Both the incentive and the participation constraint are binding, but trying to increase \( d \) in order to make the incentive constraint vanish (\( \mu_1 \to 0 \)), the imputed value to the principal of giving an extra unit of income to the agent approaches 1 for the risk-neutral and 0 for the risk-averse agent.

Therefore, for risk-neutral agents, first best can be infinitely approximated by increasing punishment. In the limit case, investing one unit of utility in the incentive scheme exactly yields one unit of utility to the principal. Marginal utility and marginal cost are the same and the outcome efficient.

**Proposition 19:** First best can also be infinitely approximated in the case where error is permitted if the agent is risk neutral and there is no bankruptcy constraint.

In the risk-averse case, if \( d \) is increased, the marginal utility of one unit invested in the incentive scheme approaches 0. Therefore, the principal will not choose \( d \to \infty \), but then, the incentive constraint will be binding and first best can no longer be achieved. To see why, one can look at the symmetric information case as a benchmark. Here, no incentive constraint has to be stipulated because compliance is assured by a forcing contract. Now, if an incentive constraint is added and it proves to be loose, it means that the same result is feasible as in the symmetric case: First best can be achieved. If it is tight, it means that the agent has to be compensated for the extra risk. Inducing the agent not to cheat comes at the price of imperfect risk sharing.

**Proposition 20:** In the case of a risk-averse agent there will be a welfare loss if error in judgement is allowed for, even if there is no bankruptcy constraint.
The described effect can be interpreted as follows: If there is no bankruptcy constraint, punishment can be made very high. If punishment is very high, the frequency of inspections ($\alpha$) can be reduced. This means that, holding monitoring cost ($\alpha c$) constant, it is possible to decrease the chance of error ($1 - p(c)$) as the accuracy $p(c)$ is an increasing function in $c$. In turn, a reduced chance of error decreases the loss due to imperfect risk sharing. Yet, there is a second effect: Holding probability of error constant, higher punishment increases welfare loss due to imperfect risk sharing if the agent is risk averse. There is consequently a direct and an indirect effect, which are countervailing in the case of agent risk averseness.

The resulting trade-off is solved by balancing monitoring cost and risk taking simultaneously on two levels: On the first level, the accuracy of monitoring is an increasing but concave function in cost per inspection ($c$). Because of concavity there comes a point where it is better to reduce the frequency of inspections and consequently economize directly on monitoring cost than to invest into accuracy, and thereby indirectly reduce the cost of imperfect risk sharing. On the second level there is another problem: With the probability of error held constant, there comes a point where the loss of imperfect risk sharing for increasing levels of punishment is higher than the savings in monitoring cost by reducing frequency.

3.1.7 Discussion

The input monitoring model of this subsection incorporates bankruptcy constraints, monitoring cost, and error in judgement.

It was shown that input monitoring can approximate first best even if it comes at a cost if there are no bankruptcy constraints and no possibility of error in judgement. If there is a bankruptcy constraint, there will be welfare loss because of direct monitoring cost and possibly efficiency wages or complete uncontractibility (no trade). The phenomenon of efficiency wages can be understood by realizing that, for purposes of incentive, provision differentials in agent pay-utility and not absolute pay-levels are relevant. Therefore, in the absence of error in judgement, increasing agent risk-averseness even helps to set up cheap incentive schemes. Yet, if error in judgement is permitted, problems of imperfect risk-sharing arise, requiring the simultaneous minimization of the cost arising from investment in the monitoring technology, the frequency of inspections and the risk premium.

For the sake of tractability, cheating was modelled as a lump sum appropriated by the agent without the consent of the principal. This leaves no place for different degrees of shirking. Therefore, in contrast to the output
monitoring model, residual loss due to lower effort in equilibrium cannot be captured. The input monitoring model distinguishes two cases: In the first case, there is no input monitoring at all and the principal expects the agent to use his full scope of shirking. Preventing the agent to shirk would be too costly. In the second case, the agent does not shirk at all, but the cost of inducing him to refrain from shirking is modelled. It becomes clear that the model set-up is coarse in this respect. It does not take into account cases wherein the principal is trying to induce the agent to refrain from shirking only to some extent. This coarseness does not mean that this case is not possible. It is just a technical consequence of the binary formulation of shirking, justified by the emphasis of this model.

3.2 Output Monitoring

3.2.1 Introduction

Error in judgement is a core part of the traditional model for output monitoring described above. Error is inevitable because of the stochastic disturbance of the production technology: If compensation depends on output, there is the danger of innocently punishing the agent. Output can be low even if effort was high because of bad luck. The risk-averse agent will want to be compensated for carrying this risk resulting in an incentive-risk trade-off. Input monitoring, on the other hand, was considered to be accurate but costly. Yet, the picture is more complex: Just as input monitoring can have a problem of imperfect risk-sharing because of the pitfalls of the monitoring process, there are situations where output monitoring is perfectly accurate and can achieve first best beyond the obvious case of a deterministic production function. This will be the case for shifting support schemes. Another problem considered will be the moral hazard with respect to risk which might arise in output monitoring schemes in the presence of bankruptcy constraints.

21. Output monitoring can achieve first best if shifting support sets are assumed and harsh enough punishment is feasible relative to the actionable portion on the joint support set.

22. The bankruptcy constraint makes variable fee schemes, which are not already asymmetrical in design de facto asymmetrical. This creates a new moral hazard problem with respect to choice of risk.

342 This means that the above assumption that density functions are defined on the same support sets is relaxed (see: IV2.4.2 ).
3.2.2 Shifting Support

It was said that output-based compensation leads to imperfect risk sharing if the agent is risk averse and the production function is stochastic. This is only true if it is assumed that choice of effort affects the density function but leaves the support set unchanged. If, however, different effort levels make the support set shift so that **ranges of outcomes do not perfectly overlap** for different effort levels, very harsh punishments can be announced. These schemes are called shifting support schemes or “boiling-in-oil contracts”\(^{343}\). First best can be achieved. The intuition is that if certain very bad outcomes can only happen if effort is lower than desired by the principal, the threat of a harsh punishment for these outcomes will induce the agent to refrain from shirking\(^{344}\).

Once again the crucial point is to exclude the possibility of error in judgement. This solution combines the **best of two worlds. Low cost** output monitoring and **no error** as in traditional input monitoring. There are, however, two problems:

\[ W = -\infty \]

![Diagram](image)

**Exhibit 7: Shifting Support Scheme\(^{345}\)**

First, as for input monitoring, the introduction of **bankruptcy** and other legal constraints is harmful to the feasibility of shifting support schemes, although

\(^{343}\) Rasmusen (1994), p. 180

\(^{344}\) This result is similar to the result of Mirrlees that step-function schemes are always better than linear incentives. It is also the basis for the idea, that if society wants to enforce compliance with rules at low cost it must impose punishments out of proportion with harm (see: Becker (1968) for criminal law and Polinsky, Che (1991) for tort law). As was mentioned above (see footnote 183), the US. Supreme Court recently ruled on this matter and drew some criticism from economists. At the same time, there are certainly moral issues involved. A quite different critical argument was mentioned in footnote 335.

\(^{345}\) Rasmusen (1994), p. 180
the problem is less severe than in the case of input monitoring, where the objective was not just to create incentives for showing effort but also to drive down monitoring cost by decreasing the frequency of inspections.

The second problem is more severe: It is difficult to imagine many situations where shifting support occurs. And if it occurs, the problem will be to predict the portion where the support sets of the two density functions do not overlap. Given the severity of the consequences this is crucial. Otherwise, the problem of imperfect risk sharing reappears. Maybe there is a portion on the joint support set where one can say that it is possible to infer with 100% certainty that low effort was exerted. Yet, these portions, which will be called “actionable portions”, tend to be very narrow. The narrower they become, the looser the bankruptcy constraint must be in order to provide appropriate incentives.

**Proposition 21: Output monitoring can achieve first best if shifting support sets are assumed and harsh enough punishment is feasible relative to the actionable portion on the joint support set (a narrow “actionable portion” requires a high bankruptcy constraint).**

### 3.2.3 Moral Hazard with respect to Risk

The bankruptcy constraint makes monitoring in general more costly. This applies for fixed and variable fee contracts. In the case of a fixed contract, limited possibility to impose punishment drives up inspection cost. In the case of a variable contract, there is the danger of unilateral increase of risk. This may require putting a cap on the upside of incentive schemes, which limits their effectiveness.

For an intuitive explanation, one can look at the case of the stochastically disturbed linear production function and a linear sharing rule. If the agent not only controls the mean but also the risk (variance) of the project, he can improve his position by increasing the risk above the optimal level. This is comparable to somebody holding a call option. He benefits if the risk of the underlying increases. For low enough expected compensation of the agent, it is preferable for him to choose a high risk project even if expected pay-off is lower. This causes potential damage to the risk-neutral principal, not only the equity holder but also the creditors. If e.g. the owner-manager also has limited liability, there could be

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346 This argument is analogous to the “fine tuning” – argument against step function schemes (see IV2.4.3).
collusion between the consultant and the client at the expense of the creditors who are the principal’s principal.\footnote{This is an example of a chain of principal-agent relationships and the problem of colluding. This can often be encountered in the real world and is a fundamental problem of corporate governance.}

Proposition 22.: The bankruptcy constraint makes variable fee schemes, which are not already asymmetric in design, de facto asymmetrical. This creates a new moral hazard problem with respect to choice of risk, which may require putting a cap on the upside of incentive schemes, limiting their effectiveness.

3.2.4 Discussion

In this Sub-Section it was shown that shifting support schemes create first best solutions for output contracting even if the agent is risk-averse. Yet, such schemes are often unrealistic because of bankruptcy constraints (legal, moral, economic) and the problem of fine tuning, already encountered earlier in the case of step functions. Another point made was the asymmetry of incentive schemes due to bankruptcy constraints, which create problems as the agent will have the incentive to choose very risky projects even if they have a lower expected value than alternative projects.

4 Transaction Cost, Bonding, Distortion

4.1 Transaction Cost and Bonding

The following propositions will be motivated in this subsection:

23. Direct transaction costs arise for input monitoring and for output monitoring, though it will often be plausible that they are lower for output monitoring.

24. Indirect transaction costs arise from provisions which are designed to lower direct transaction costs. The goal is to rationalize monitoring or to directly influence the agent’s disutility function. This leads to inefficiencies because production technology is prescribed from top to bottom in a way that is known to be inefficient. In addition, innovation from bottom to top is stifled. This is a distortive effect.
One of the parameters of the input monitoring model was the **direct transaction cost** for putting in place the monitoring device. Inspections have to be carried out either simultaneously or ex post. Not only input monitoring but also output monitoring may cause such extra transaction cost, though it will usually be lower: Required output has to be defined and provisions have to be made to record output thus defined, sometimes resulting in extra accounting expenses.

Less obviously, there are **indirect costs** of monitoring: **Bonding costs** arise when the agents have to abide by certain strict rules. Such measures can have two purposes: Either they **influence the disutility function** by reducing distraction, or they **rationalize monitoring**. Surfing the internet, phoning privately, walking through the park during office hours might be prohibited. If a consultant is required to work on site in an office assigned to him, his disutility function is influenced by reducing distraction (it is not very interesting to sit in the office looking out of the window, while it may be very attractive to sit down and watch TV). In addition, it is easier to monitor his actions in the office than in the field. Another example is a private investor instructing his actions in the office than in the field. Another example is a private investor instructing his banker not to make certain kinds of investments. He may forgo profit opportunities (e.g. by not allowing him to exceed a certain leverage), but also protects himself against the hazards of excessive risk taking. This actually comes close to **prescribing production technology** from top to bottom, potentially stifling innovation and forcing people to use inefficient technology. On the other hand, it helps to circumvent situations that are difficult to monitor. Bonding is thus conditioned by the monitoring device but it also has features of **distortion**, which will be treated below.

### 4.2 Distortion

#### 4.2.1 Introduction

The discussion so far focused on parties who want to contract on effort but may decide to contract on output. This is because they expect the advantage of observability and verifiability to outweigh the potential disadvantage of imperfect risk sharing\(^\text{348}\), due to the increased importance of the external factor. In fact, the above models of output monitoring and input monitoring serve to derive **optimal contracts stipulating contingencies upstream in the case of input monitoring** and **downstream in the case of output monitoring**. In a next step the two **optimal contracts can be compared**. In a situation where the optimal output monitoring contract dominates the optimal input monitoring contract, more

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\(^{348}\) If the agent is relatively more risk-averse.
downstream performance measures will be chosen and vice versa. As a by-product of the analysis, much is said about how to specify the resulting contract.

But, so far, one important problem was ignored: If the principal switches to alternative measures of performance because total contribution cannot be contracted upon (or only at prohibitive cost), there will not only be the potential problem of imperfect risk-sharing: There will also be a tension between what the principal desires and what the agent is rewarded for. This problem, called distortion, was highlighted by Kerr, when he described the "folly of rewarding A while hoping for B". Kerr attributes this problem to the "fascination with objective performance measures". Therefore, "if you can't measure what you can, you end up wanting by what you can measure".

But, the discussion above showed that there is more to this problem than a simple psychological trap, easily avoided by cool-headed rational thinking. What is wanted just may not be contractible. If this problem is unattended to, many potential gains of trade cannot be realised because people will not trade. So, if a related alternative performance measure can be found that can be contracted upon, some of the gains can be realised by going for this second best solution. So far, imperfect risk sharing, residual loss due to shirking, and direct cost of the monitoring mechanism were mentioned. Now, another source of loss must be added: distortion. The following propositions will be derived:

25. If the weights measuring the effect of the different actions on the performance measure are generally higher than the weights reflecting the contribution of these actions to the principal's value, the bonus rate will be small.

26. If the incentives are well aligned with the ultimate goal, distortion will be low and the bonus rate will be high.

Kerr (1975)

Gibbons (2001), p. 4
4.2.2 The Model

If total contribution \( y \), or “everything the principal cares about except for wages”\(^{351} \) is not contractible, the agent’s incentive will depend on an alternative performance measure \( p \). “The essence of the incentive problem, is the divergence between the agent’s incentive to increase \( p \) and the principal’s desire to increase \( y \)”\(^{352} \). To clarify the effect, a model will be introduced.

If \( y = a + \varepsilon \) and \( p = a + \phi \), and the contract specifying compensation is \( w = s + bp \), there will be no such distortion. The agent, by trying to increase his compensation also increases the principal’s utility \( y \). It can, however, be imagined that two actions (or “tasks”) are required in order to promote the principal’s interest. Consider a client who might be interested in cutting his cost in order to enhance his long-term profit perspectives. Thus, the tricky task for the consultant is to cut costs without decreasing the capability of the client to produce valuable products and services to his customers; it is normally straightforward to observe and verify cost cutting while less obvious to assess the implications on the company’s capabilities, which will show much later, if at all. To formalize this situation, \( y \) is modelled to depend on two tasks \( y = a_1 + a_2 + \varepsilon \) (in the example \( a_1 \) would be cost cutting and \( a_2 \) the observance of the restriction that capabilities must be maintained). The performance measure will be \( p = a_1 + \phi \). It can easily be seen that the agent’s incentive to perform one task or the other depends on the way \( a_1 \) and \( a_2 \) affect the performance measure \( p \) and on the bonus rate \( b \). Therefore the agent will promote \( a_1 \), but not \( a_2 \).

Another case is where \( y = a_1 + \varepsilon \) and \( p = a_1 + a_2 + \phi \). The agent will have an incentive to perform both tasks \( a_1 \) and \( a_2 \) although \( a_2 \) creates no value at all for the principal. An example would be the problem of “impression management”. Agents will sometimes devote considerable resources to improve the impression the principal gets from them instead of pushing forward with the real task. Even if the agent does not know by which criterion he is evaluated, he will overemphasize highly visible tasks. There are some performance criteria that are good indicators of performance as long as they are not used as incentives. In order to evaluate a teacher, students’ performance on standardized tests may be a good indicator. As soon, however, as this is used as an incentive, teachers will try to “teach to the test”. The principal (in this case the parents or the government) may fail to get the good education for their children they ultimately desire.

\(^{351} \) Gibbons (2001), p. 5
\(^{352} \) Gibbons (2001)
The extreme case, \( y = a_1 + \varepsilon \) and \( p = a_2 + \phi \), is where the agent only performs \( a_2 \), not creating any value at all. The models dealing with these problems are called “multi-task models.” A tractable example will be presented in the following:

It is assumed that the value created to the principal is given by:

\[
y = f_1 a_1 + f_2 a_2 + \ldots + f_n a_n + \varepsilon = \bar{f}a + \varepsilon.
\] (7.1)

And the measured performance is given by:

\[
p = g_1 a_1 + g_2 a_2 + \ldots + g_n a_n + \phi = g\bar{a} + \phi.
\] (7.2)

The principal offers the agent a linear contract contingent on \( p \),

\[
w = bp + s.
\] (7.3)

The agent accepts the contract if compensation is high enough to cover his costs which are assumed to be:

\[
c(\bar{a}) = 1/2\bar{a}^2.
\] (7.4)

He will choose his actions \( \bar{a} \) unobserved by the principal. The principal determines \( p \) and pays the compensation as specified by the contract. Even if the principal cannot observe \( \bar{a} \), given the terms of the contract he can perfectly predict the choice of the utility maximizing rational agent. Against this backdrop, he has to choose the contract terms maximizing his own utility. This is the familiar agency model where the principal maximizes his expected pay-off subject to an incentive and a participation constraint. For simplicity it is assumed that both principal and agent are risk neutral, so that both maximize expected value \( E_p, E_A \). The maximization problem can therefore be set up as follows:

\[
\max_{b,s,\bar{a}} \bar{f}a - b(g\bar{a}) - s
\] (7.5)

---

353 These models were originated by Holmström/Milgrom (1991)

354 The model presented is a slightly more general version than the model presented by Gibbons (2001). Gibbons refers to more elaborate models: see Feltham and Xie (1994), Kulp, Datar, and Lambert (1999), and Baker (2000).

355 Whether or not he observes \( y \) is irrelevant in the one-shot relationship.
s.t. IC: \( \bar{a}^* \in \arg \max_{\bar{a}} b(\bar{g}\bar{a}) + s - \frac{1}{2} \bar{a}^2 \) \hspace{1cm} (7.6)

PC: \( b(\bar{g}\bar{a}) + s - \frac{1}{2} \bar{a}^2 \geq 0 \) \hspace{1cm} (7.7)

The incentive constraint is a maximization problem. Therefore, the first-order condition must hold.

\[ \text{Grad} (E_A) = 0 \iff b\bar{g} - \bar{a} = 0 \] \hspace{1cm} (7.8)

Replacing the maximization problem of the incentive constraint by the first-order condition requires it not only to be necessary but also sufficient. It can easily be seen that:

\[ \frac{\partial E_A^2}{\partial a_i^2} = -1 \text{ and } \frac{\partial E_A^2}{\partial a_i \partial a_j} (i \neq j) = 0 \] \hspace{1cm} (7.9)

Therefore, the quadratic form is negative definite and \( E_A \) is concave. The first-order condition can therefore replace the maximization problem of the incentive constraint.

Inserting (7.8) and restating the maximization problem gives:

\[
\max_{b,s,a} \bar{a} - b(\bar{g}\bar{a}) - s
\]

s.t. IC: \( b\bar{g} - \bar{a} = 0 \iff \bar{a} = b\bar{g} \) \hspace{1cm} (7.6)'

PC: \( b(\bar{g}\bar{a}) + s - \frac{1}{2} \bar{a}^2 = 0 \) \hspace{1cm} (7.7)

Solving (7.6)' for \( \bar{a} \) and inserting into (7.7) gives:

\[ b^2\bar{g}^2 + s - \frac{1}{2} b^2\bar{g}^2 = 0 \] \hspace{1cm} (7.7)'

Solving (7.7) for \( s \) yields:

\[ s = -\frac{1}{2} b^2\bar{g}^2 \] \hspace{1cm} (7.7)''
Inserting (7.7)′′ and (7.6)′ into (7.5) gives:

\[ b\vec{f}g - b^2\vec{g}^2 + \frac{1}{2} b^2\vec{g}^2 = b\vec{f}g - \frac{1}{2} b^2\vec{g}^2 \]  (7.5)′

The first-order condition for an optimal \( b \) is therefore:

\[ \vec{f}g - b^*\vec{g}^2 = 0 \]  (7.10)

Solving for \( b^* \) yields:

\[ b^* = \frac{\vec{f}g}{\vec{g}^2} \]  (7.10)′

As \( \vec{f}g = |f||g|\cos \varphi \) and \( |\vec{g}| = \sqrt{\vec{g}^2} \Leftrightarrow \vec{g}^2 = |\vec{g}|^2 \):

\[ b^* = \frac{|f||g|}{|g|^2} \cos \varphi = \frac{|f|}{|g|} \cos \varphi, \]  (7.11)

where \( \varphi \) is the angle between the vectors \( \vec{f} \) and \( \vec{g} \) and \( |f|, |g| \) the length of the vectors.

It becomes clear from the model that the optimal bonus rate depends on two important factors: scaling and alignment\(^{356}\).

If the weights\(^{357}\) \( \vec{g} \) measuring the effect of the different actions on the performance measure \( p \) are generally higher than the weights \( \vec{f} \) reflecting the contribution of these actions to the principal's value \( y \), it means that \( p \) is relatively more sensitive to higher levels of action than \( y \). One would therefore expect the bonus rate to be small. This is exactly what is expressed in the model.

**Proposition 25:** If the weights measuring the effect of the different actions on the performance measure are generally higher than the weights reflecting the contribution of these actions to the principal's value, the bonus rate will be small.

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\(^{356}\) Gibbons (2001) p. 7

\(^{357}\) By analogy one could speak of marginal products
Besides the overall scale of the weights, a relevant feature is the extent to which the weights of y and p have a similar pattern. If the pattern is similar, the distortive effect by “rewarding A, while hoping for B” will be small. This is because there will not be many instances where a particular action has a strong effect on p but not on y. For intuitive illustration, consider the graphical interpretation of vectors  \( \mathbf{f} \) and  \( \mathbf{g} \), representing the weights on y and p respectively. In fact, the cosine of the angle between them summarizes the extent of pattern similarity. If the angle between them is small (and cosine will be high), they roughly point in the same direction. They are well “aligned”. In these cases, one would expect the bonus rate to be rather high since distortive effects are low. This is exactly what is shown in the model: The lower the angle, the higher \( \cos \varphi \) and therefore the higher the bonus rate.

**Proposition 26:** If the incentives are well aligned with the ultimate goal, distortion will be low and the bonus rate will be high.

### 4.2.3 Discussion

Distortion arises if there is a tension between what the principal wants and what the agent is rewarded for. Often it is not possible to eliminate this tension. What the principal wants just might not be contractible. As was shown in this subsection, distortion can be divided into two components: scaling and alignment. Scaling refers to the relative sensitivity of the two measures to changes in the drivers, and alignment to the similarity of driver patterns. If a university rewards a scientist (by promotion or resources) according to the number of articles published during a certain period of time (driver), there will be a problem of alignment if the university cares about both quantity and quality. Indeed, the researcher would have the incentive to publish many articles in low quality journals. If no ranking of journals to account for quality is available, rewards should not depend too much on the number of published articles. Now, considering different departments it could be that a typical researcher in, say, marketing has 5 times as many publications than a typical researcher in, say, mathematics. If the basis for the bonus is the number of published articles, a scaling argument suggests the bonus rate for marketing researchers to be one-fifth of the bonus rate for mathematicians.

As will be elaborated later in more detail, it is obvious that there is a conflict with the risk-incentive trade-off. This model suggests that the bonus rate should be high if the observed variable is relatively undisturbed\(^{358}\). This will be the case

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\(^{358}\) see Proposition 4
for upstream parameters, but these will be the most distorted which suggests that
the bonus rate should be low.

5 Dynamic Extensions

5.1 Introduction

Up to now, the analysis has been largely one period. Only in one instance did a
dynamic idea sneak into the argument; namely, when it was argued that non-linear
incentive schemes – in particular the step function scheme – create path-dependent incentives\(^{359}\). This was taken as a favourable feature of linear incentive schemes. Yet, there is a more general point to be made about dynamic
extensions.

Many traditional models of contract theory are one period. The subject of
this Chapter will be to analyse the effect of time on contracts. The starting point of
this discussion is the often stated thesis that time can resolve incentive issues that
arise in one-shot relationships costlessly, or at least can significantly reduce
incentive costs\(^{360}\). Four models will be presented here to discuss this question. The
first model deals with the advantage of long-term contracts over short-term
contracts. Time allows lowering the cost of incentives by reducing imperfect risk
sharing of output-based contracts (5.2). The second and third models deal with
situations wherein relational contracts solve problems of enforcement. The theory
of supergames will be used to argue that time may sustain contracts with
otherwise desirable properties, which would not be feasible in a one-shot
relationship. This is the case where contract parameters are observable but not
verifiable (5.3). The fourth model introduces career concerns which induce the
agent to exert effort, although choice of effort cannot be contracted on. It will be
shown that an implicit contract links the agent’s current choice of effort to future
pay-off. (5.4). In conclusion, it will be argued that the thesis that time solves
incentive issues costlessly cannot be generally upheld. Time merely alters and
enriches the insights from one-period models: Conclusions from the one-period
models are not necessarily valid in the multi-period settings.

\(^{359}\) see Sub-Section IV2.4.3

\(^{360}\) Fama (1980)
5.2 Income smoothing

5.2.1 Introduction

Whenever parties are unable to contract on what they are really interested in, they are forced to switch to alternative performance measures. Output monitoring was shown to be relatively cheap and undistorted, but input monitoring created less exposure to the external factor leading to better risk sharing.

The Mirrlees argument\textsuperscript{361} on the superiority of step functions over linear incentive schemes and the "shifting support" argument both attempted to combine the best of two worlds: Perfect risk sharing in the presence of output monitoring. These solutions were mainly attacked on practical grounds. It could be established that such schemes were an extreme case of fine tuning\textsuperscript{362}.

Time alters this argument. If many periods are observed, the law of large numbers filters out uncertainty and it becomes easier to distinguish shirking from bad luck. The intuition is simple: In a one-shot relationship, if the project fails and the agent is punished, there is a considerable risk that he is punished innocently. If a project fails repeatedly, the risk of this being due to bad luck decreases. Thus, the cost of imperfect risk sharing is lower than in the multi-period case\textsuperscript{363}. In practice, the principal can offer the principal a long term contract wherein the decision on bonus or punishment is made towards the end of the contract. Until then he is paid a low but regular income. If the principal can commit to such a scheme, the agent knows that if he is not shirking he will receive the bonus. In this Section, the following proposition will be illustrated:

27. Long term contracts can provide cheaper incentives than a sequence of short term contracts if the agent has saving and borrowing constraints.

5.2.2 The Model

In order to further illustrate this argument, a simple model is constructed: It is assumed, that the agent can choose two levels of effort \( a_1 \) and \( a_h \) representing the mean of a normal distribution \( \bar{Y}_1 = a_1 + \varepsilon \) and \( \bar{Y}_h = a_h + \varepsilon \), where \( \varepsilon \sim N(0,\sigma) \), \( a_h > a_1 \). In the one-shot relationship the distribution of outcomes contingent on effort is therefore:

\textsuperscript{361} Mirrlees (1974)
\textsuperscript{362} Hart, Holmström (1987), S. 90/91
\textsuperscript{363} Radner (1981)
\[ Y_i \sim N(a_i, \sigma) \]
\[ Y_h \sim N(a_h, \sigma) \]  

(12.1)

The n-period relationship is modelled as the sum of n identically distributed, independent random variables (actions are uncorrelated):

\[ Y'_i = nY_i \]
\[ Y'_h = nY_h \]  

(12.2)

Applying the law of large numbers, the distribution of \( Y'_i \) and \( Y'_h \) respectively can be calculated to be:

\[ Y'_i \sim N\left( na_i, \sqrt{n}\sigma_i \right) \]
\[ Y'_h \sim N\left( na_h, \sqrt{n}\sigma_h \right) \]  

(12.3)

Obviously, the distance of means increases proportionately, while the standard error increases less than proportionately with time. Thus, separation between the two distributions becomes better. This will be shown in Exhibit 8, where the density functions of shirking vs. non-shirking in the one-shot and in the multi-period case are depicted:

In the area where the chance of punishing the honest agent is infinitesimal, harsh punishments can be inflicted without additional monitoring or review procedure. The portions where this is the case are called “actionable range” in Exhibit 8. It becomes clear that the shirking agent expects a probability of punishment of about 5% in the one-shot relationship under such a scheme, whereas he expects a probability of punishment of about 50% in the long-term relationship. 5% may be enough if punishment can be high enough, but bankruptcy constraints are likely to render such a scheme unfeasible. Therefore, either imperfect risk-sharing has to be accepted or ex post monitoring (in the form of a revision process) has to be introduced.

The cost of monitoring is determined by the monitoring technology and the probability that the monitoring process is triggered. It will be assumed for simplicity that ex post monitoring is accurate and comes at a given cost. So, the probability of revision taking place is the only cost driver. For concreteness, it is assumed that punishment can be made high enough to make the agent choose “not shirking” if the probability of detection is 66%. It becomes clear that the monitoring process is triggered in 50% of the cases (see area ABC) in the one-
shot relationship and in about 8% of the cases (see area DEF) in the multi-period relationship. The range of outcomes triggering the revision process is referred to in Exhibit 8 as “control range”.

\[ \bar{Y}_c \sim N(2,5) \]
\[ \bar{Y}_p \sim N(7,5) \]
\[ n=8 \]

**Exhibit 8: Shirking/Non-Shirking: One-shot vs. Long-term**

If it is assumed that in the one-shot relationship the actionable range will never be sufficiently wide to implement first best at realistic bankruptcy constraints, there will have to be an ex-post monitoring process. If there is a chance of error, punishment cannot be too high. The control range will have to be rather wide relative to the relevant support set (if “not shirking” is implemented, the relevant support set is that of “not shirking”). In the multi-period case, the “actionable range” may be wide enough to achieve first best at realistic
punishment levels. If additional monitoring is needed to avoid imperfect risk-sharing, the control range will be small relative to the relevant support set. Therefore, in the multi-period case "punishment can be made harsher and the control range tighter"\textsuperscript{364}.

5.2.3 Discussion

Thus, shirking can be dealt with more cheaply in the multi-period relationship, because the principal can engage in income smoothing for the agent. This, however, will only be of value to the agent if he has saving and borrowing restrictions which is not necessarily the case\textsuperscript{365}. Still, information problems of an outside party suggest that the principal of the primary relationship will often be the privileged counterparty for such transactions. Therefore:

\textit{Proposition 27: Long term contracts can provide cheaper incentives than a sequence of short term contracts if the agent has saving and borrowing constraints.}

One should understand the nature of this transaction in order to preclude any misunderstanding. Saving and borrowing enables people to shift the present value of their business relationships in time. It is just the possibility for the agent to borrow if he knows that by bad luck he received less as he would normally receive and to save if he has a windfall profit higher than his effort would normally justify. This is no case of insurance. There is almost certainty about the present value, because in equilibrium the agent will not shirk, which will ultimately be seen by the principal\textsuperscript{366}.

\begin{itemize}
    \item \textsuperscript{364} Holmström, Hart (1987)
    \item \textsuperscript{365} Allen (1985)
    \item \textsuperscript{366} Of course, if one allows for the agent to default then there might be a moral hazard if too much borrowing is permitted and the incentive scheme breaks down. This will also happen if insurance in its proper sense is possible for the agent. In this case, the agent will receive the same utility whatever the circumstances. In such a situation, there would be no incentives for the agents to exert effort. But normally, such insurance would never be offered. A well-known case where this might happen nevertheless is if the agent can securitize the present value of his claims from his business relationship and sell them. This happens when managers receiving stock options for incentive reasons sell these stock-options in the market in order to reduce Exposure, which is why there are normally restrictions to such sales in stock-option schemes.
\end{itemize}
5.3 Reputation Effects in Supergames

5.3.1 Introduction

In the last Section, time was built into an explicit long term contract and allowed to lower the cost of incentives by reducing imperfect risk sharing of output-based contracts. Output-based contracts were an answer to the uncontractability of input parameters.

It was already argued that contractibility presupposes the knowledge of the production function, observability and verifiability. An especially interesting case is where parties can observe a performance measure that will not be verifiable by a third party like a court. There are certain situations wherein a self-enforcing mechanism – also referred to as an implicit contract – exists, sustaining a contract based on such subjective performance measures. The fundamental reasoning for these mechanisms is that if one of the parties makes a promise, it must be able to commit to this promise. Otherwise, the promise is worthless and cannot create incentives. In other words, it must be clear that at the moment when the party will have to make good on its promise it must be in its interest to do so. Otherwise, it can hold up the other party. Thus, the centre of interest is the decision rule of the party.

Consider, for instance, a situation where effort is observable but cannot be objectively verified. The principal cannot commit to paying a bonus contingent on effort because it will always be in his interest to renege later. So, maybe he commits on something else that constrains his future action space in such a way that it will be in his interest to make good on his promise. The tournament mechanism is a case in point. The principal facing many agents commits based on the total amount of bonuses paid out. By taking away the option of saving money by reneging, he can also credibly commit to paying the bonus as promised if only an infinitesimal preference for honesty is assumed. In this Section, it is shown that long term relationships actually are able to create circumstances in which parties find it easier to commit.

The basic intuition is simple: If one party has experienced that the other party acted opportunistically, it will stop doing business with this party. However, if the other party values the ongoing trade relationship, it will, anticipating this decision, not let its business partner down in the first place. It is therefore argued

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367 see Gibbons (2001)
368 see Lazear (1981)
that long term relationships can in some circumstances support contracts that may otherwise not be feasible by reputation effects created between the parties\textsuperscript{369}.

In the following subsections, two models will be presented: The first model (5.3.2) deals with a situation where effort is observable but not objectively verifiable. It was argued that in such cases, parties will switch to output-based bonus contracts. Yet, reputation effects may make a flat fee contract based on observed effort feasible. This is because the agent will not engage in shirking because his reputation is at stake.

The second model (5.3.3) deals with a situation where effort is not observable. Parties therefore switch to output monitoring, but now it is assumed that it is output which cannot be objectively verified, although it is observable to both parties. In such a case, it can be argued that it was impossible to create incentives because the principal cannot commit on the bonus payment. Again, reputation effects can make such an arrangement feasible because the principal might refrain from reneging because of reputation concerns. The following propositions will be derived:

28. It can be seen that the agent will be less likely to renege if gains of trade are high (low $\beta$), the agent’s discount rate is low (high $\delta$), the agent’s expected growth rate for the value of the trade relationship is high (high $\phi$) and the bargaining power of the agent is high (high $\gamma$). If the growth rate is zero ($\phi = 1$), it can be seen that condition (12.8)’ always holds for a discount rate approaching 0 ($\delta \to 1$) if the agent gets at least a tiny fraction of the gains of trade, as was stipulated in the assumptions ($\gamma \in (\beta,1]$). In this case, first best will always be feasible.

29. Reputation effects are more likely to sustain a bonus contract if the value of the trade relationship ($\Delta = H - L$) is high and the principal’s discount rate ($\tau$) is low.

30. The strong assumption of indefinite repetition can be relaxed by assuming uncertainty with respect to game’s conclusion.

\textsuperscript{369} The classic reference is Bull (1987)
5.3.2 Observable but Uncontractible Effort

The basic idea of reputation effects is that present action influences not only pay-off in the current period, but also in future periods\(^{370}\). Thus, in any period the agent has to take into account the payment of this period and of the following \(t > \tau\) periods. It is assumed that the agent can decide whether to choose high or low effort \((a_L, a_H)\), which stochastically determines output \((y_i = a_i + \epsilon_i\) with \(i = L, H)\). The disutility of effort is assumed to be a linear function of effort \((\beta a_L, \beta a_H)\)^{371}. The principal will commit to paying a flat fee that will be between the expected cost to the agent \((\beta a^e)\) and the expected value of output \((a^e)\). The exact distribution of the gains of trade will be determined by bargaining, and depend on the bargaining power\(^{372}\) \((\gamma)\):

\[
\beta a^e < \gamma a^e \leq a^e \Rightarrow \gamma \in (\beta, 1] \quad (12.4)
\]

Profit for the agent will therefore be:

\[
\gamma a^e - \beta a_i, \quad (12.5)
\]

which depends on the principal’s expectations \((a^e)\) and on the agent’s choice of effort \((a_i)\). Independent of whether or not the principal agrees to a low or a high flat fee, the agent will profit from choosing low effort. Therefore, the principal will have low expectations \((a^e = a_L)\). The profit for the agent will thus be \(a_L (\gamma - \beta)\). Because \(a_L (\gamma - \beta) < a_H (\gamma - \beta)\), profit would be higher for the agent if he could commit to choosing high effort \((a_H)\). This is impossible in the one-shot relationship.

However, if the parties are playing a repeated version of this one-shot game this might change. It is assumed that the principal plays a trigger strategy. He expects the agent to choose high effort until he observes that he is choosing low effort. In this case he will assume low effort forever after.

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370 This model is inspired by a model of Bester in an unpublished script.
371 Disutility is usually modelled as a convex function in effort. In this model, there is another focus and a linear disutility function is assumed for ease of exposition.
372 There are some models (e.g. Rubinstein 1982, see Kreps 1990, 556n) where the outcome of bargaining depends on the process of bargaining. An alternative to a bargaining solution would be to assume that the market mechanism determines a market price. It is common e.g. to assume that many principals compete against each other, driving the profits down to zero. As it is intended to apply the models to the client-consultant relationship where services are normally very specific to the relationship, a bargaining approach is taken.
The agent’s discount factor\(^{373}\) is assumed to be \(\delta\) and the growth rate of the expected gains of trade \(\phi\). The growth rate is introduced to model the agent’s expectation that the business relationship with the client will increase or decrease in value over time. Then the agent’s pay-off for choosing high effort \((a_H)\) in period \(\tau\) is:

\[
\sum_{t=0}^{\infty} \delta^t \phi^{t+\tau} (\gamma - \beta) a_H
\]

Doing some algebra\(^{374}\) this can be written as:

\[
\phi^* \frac{a_H (\gamma - \beta)}{1 - \delta \phi} \quad \text{for } \delta \phi \neq 1
\]

On the other hand, the pay-off from defecting will result in a higher pay-off in the first period but in lower pay-offs forever after:

\[
\phi^* (\gamma a_H - \beta a_L) + \sum_{t=1}^{\infty} \delta^t \phi^{t+\tau} (\gamma - \beta) a_L
\]

Again, doing some algebra\(^{375}\) yields:

\[
\phi^* \left[ (\gamma (a_H - a_L) + \frac{(\gamma - \beta) a_L}{1 - \delta \phi}) \right]
\]

The agent will choose high effort if pay-off \((12.6)^*\) is bigger than pay-off \((12.7)^*\):

\[
\phi^* \frac{a_H (\gamma - \beta)}{1 - \delta \phi} > \phi^* \left[ (\gamma (a_H - a_L) + \frac{(\gamma - \beta) a_L}{1 - \delta \phi}) \right]
\]

This condition can be simplified\(^{376}\) to:

\[
\beta < \delta \phi \gamma
\]

---

\(^{373}\) The agent’s reputation is at stake here.

\(^{374}\) See Mathematical Appendix at the end of this Paragraph.

\(^{375}\) See Mathematical Appendix at the end of this Paragraph.

\(^{376}\) See Mathematical Appendix at the end of this Paragraph.
Proposition 28: It can be seen that the agent will be less likely to renege if gains of trade are high (low $\beta$), the agent’s discount rate is low (high $\delta$), the agent’s expected growth rate for the value of the trade relationship is high (high $\phi$) and the bargaining power of the agent is high (high $\gamma$). If the growth rate is zero ($\phi = 1$), it can be seen that condition (12.8) always holds for a discount rate approaching 0 ($\delta \rightarrow 1$) if the agent gets at least a tiny fraction of the gains of trade, as was stipulated in the assumptions ($\gamma \in (\beta, 1]$). In this case, first best will always be feasible.

Mathematical Appendix

Footnote 374:

$$\sum_{t=0}^{\infty} \delta^t \phi^{t+r} (\gamma - \beta) a_H = \phi^r \sum_{t=0}^{\infty} \delta^t \phi^t (\gamma - \beta) a_H$$

$$= a_H (\gamma - \beta) \phi^r \sum_{t=0}^{\infty} (\delta \phi)^t = \phi^r \frac{a_H (\gamma - \beta)}{1 - \delta \phi}$$

Footnote 375:

$$\phi^r (\gamma a_H - \beta a_L) + \sum_{t=1}^{\infty} \delta^t \phi^{t+r} (\gamma - \beta) a_L$$

$$= \phi^r (\gamma a_H - \beta a_L) + \phi^r (\gamma - \beta) a_L \left[ \frac{1}{1 - \delta \phi} - 1 \right]$$

$$= \phi^r (\gamma a_H - \beta a_L - \gamma a_L + \beta a_L) + \phi^r \frac{(\gamma - \beta) a_L}{1 - \delta \phi}$$

$$= \phi^r \left[ \gamma (a_H - a_L) + \frac{(\gamma - \beta) a_L}{1 - \delta \phi} \right]$$

Footnote 376:

$$\phi^r \frac{a_H (\gamma - \beta)}{1 - \delta \phi} > \phi^r \left[ \gamma (a_H - a_L) + \frac{(\gamma - \beta) a_L}{1 - \delta \phi} \right]$$
5.3.3 Observable but Uncontractible Output

If effort is uncontractible, parties might switch to output-based compensation. However, output could be observable to the parties while not objectively verifiable. This raises the possibility that the principal reneges on the promised bonus. He might, however, decide not to renge for reputation concerns.

Two possible levels of outcomes are assumed: “low outcome (L)” and “high outcome (H)”. Effort \((a \in [0,1])\) is interpreted as the probability of “high outcome (H)”. Therefore, expected outcome is:

\[
aH + (1-a)L = L + a(H-L)
\]

(12.9)

The efficient effort choice, maximizing joint utility and used as a benchmark is:

\[
\max_a L + a(H-L) - c(a)
\]

(12.10)

Taking derivatives, one can write:

\[
b^* = c'(a) = H - L
\]

(12.11)
The principal promises the agent to pay a base salary \( s \) plus a bonus \( b \) in the event that he observes high outcome \( (H) \). If the principal honours the contract when high outcome is observed, his pay-off will be:

\[
H - s - b
\]

(12.12)

If he reneges his pay-off will be:

\[
H - s
\]

(12.13)

If the agent expects the principal to honour his promise his expected pay-off will be:

\[
s + ab - c(a)
\]

(12.14)

If he expects the principal to renege:

\[
s - c(a)
\]

(12.15)

The disutility of effort for the agent is a strictly increasing convex function in \( a \):

\[
c(\cdot) > 0, \ c'(\cdot) > 0, \ c''(\cdot) > 0 \lim_{a \to 1} c(a) = \"\infty\"
\]

(12.16)

It is assumed that the agent is playing a trigger strategy: he expects the principal to honour his contract until he defects. In this case he assumes defecting forever after, but if the agent expects that he will not be paid a bonus, he will not exert effort \( (a = 0) \) and outcome will be low in certainty. In this case the principal knows that his pay-off will be:

\[
L - s
\]

(12.17)

An additional assumption will be included here: The project will only be profitable in the event of high outcome:

\[
L - c(0) - w_a < 0,
\]

(12.18)

where \( w_a \) is the agent’s reservation utility. Consequently, the principal will not be willing to offer a contract \( s \geq c(0) - w_a \). But, any other contract would
violate the agent’s participation constraint and would therefore not be accepted. No trade takes place and pay-off is zero for both parties.

Thus, if the principal defects in period $\tau$, he will have a higher pay-off in this period but zero pay-off for periods $t > \tau$. If he does not renege, his expected pay-off for periods $t > \tau$ will be $L + a(H - L) - s - ab$.

Therefore, reputation concerns will induce the principal not to renege, if the following condition holds:

$$(H - s) + 0 \leq (H - s - b) + \frac{L + a(H - L) - s - ab}{r}$$  \hspace{1cm} (12.19)

The agent’s incentive constraint if he thinks that the principal will honour the contract is:

$$a \in \arg \max_{a'} s + a' b - c(a')$$  \hspace{1cm} (12.20)

Replacing the maximization problem by the first order condition (which can be shown to be necessary and sufficient) yields:

$$b = c'(a) \Rightarrow a = a^*(b)$$  \hspace{1cm} (12.21)

Using (12.21), agent’s participation constraint is given by:

$$s + a^*(b) b - c(a^*(b)) \geq w_a$$  \hspace{1cm} (12.22)

Using (12.22) and rearranging, (12.19) can be written as

$$b \leq \frac{a^*(b)[H - L] + L - c(a^*(b)) - \bar{W}}{r}$$  \hspace{1cm} (12.23)

or $b \leq \frac{V(b)}{r}$

*Exhibit 9* shows this relationship (for $c(a) = -a/a - 1$, and $L - w = 0$). The shaded area represents the contracts satisfying the reneging constraint condition (12.23) i.e. contracts that can be sustained by reputation concerns of the principal. The function $b^*(a)$ represents the efficient bonus rate maximizing joint utility (see (12.11)).
It can be seen that, for a given differential between high and low outcome \( \Delta = H - L \), first best incentives are possible at a low discount rate (Exhibit 9 (a)). From condition (12.11), it follows that the efficient bonus rate \( b^* \) equals the differential between high and low outcome \( \Delta \). If the principal is offering \( b^* \), the agent will perform the efficient effort \( a^* \). He anticipates that the principal will honour his promise because the reneging constraint is loose (point A is within the shaded area). In other words: As not honouring the contract would hurt his long-term interests, the principal can provide efficient incentives which are credible to the agent. As the discount rate increases, this may no longer be the case.

In Exhibit 9 (b), the principal cannot commit on the efficient bonus rate \( b^* \). If the principal was to offer \( b^* \) the agent believes that he will renege on his promise (point A is outside the shaded area) and will therefore not exert any effort. The principal, anticipating that, will try to induce the highest effort level \( a' < a^* \), not violating the incentive constraint. At the given outcome differential \( \Delta \), \( a' \) is constructed by moving from A parallel to the abscissa until reaching B, which is just within the shaded area, and then projecting down onto the axis to
find $a'$. The second best bonus level is found by moving upward to C, which lies on $b'(a)$. Moving to the ordinate gives the bonus level $b' < b^*$, which is needed to induce $a'$. Thus, second best incentives can be created. If the discount rate becomes very high, there may not be any bonus rate, which is small enough to satisfy the reneging constraint. This is the case in Exhibit 9 (c). No trade will take place.

It can also be seen that if the discount rate is close to zero, first best incentives can always be provided at any output differential $\Delta$ (even if there is a small value to the trading relationship); but if the discount rate increases, first best contracts may still be possible if there for very high $\Delta$.

**Proposition 29:** Reputation effects are more likely to sustain a bonus contract if the value of the trade relationship ($\Delta = H - L$) is high and the principal's discount rate ($r$) is low.

### 5.3.4 Reinterpretation of the Discount Rate

In both models, one assumption was infinite repetition of the basic one period game in order to create reputation effects. Assuming that the relationship ends after $t$ periods, clearly in the last period $t$, the dominant strategy will be to defect; but then there will be no value to the trade relationship in the last period, meaning that also in period $t-1$ the optimal strategy will be to defect, etc. The game will thus unravel backwards and the argument breaks down. Indefinite repetition, however, seems to be a very strong assumption.

Fortunately, the models can be saved if it is assumed that the game is not infinitely repeated but instead concludes at an uncertain date. In fact, the discount factor $\delta$ in Section 5.3.2 and the discount rate $r$ in Section 5.3.3 can be reinterpreted as a combination of the actual discount factor [rate] and the probability of the game ending after each period played. If the probability of ending is $q$, the probability of the game continuing in the next period is $1 - q$. Thus, if the actual discount factor [rate] of the party whose reputation is at stake is $\mu \ [s]$, the present value of a regular pay-off $V$ is given by:

---

378 In fact, for $a^* = 0$, no contract will be offered. The intercept (D) is just the bonus that would minimize losses to the principal if he offered a contract.

379 Gibbons (2001)
\[
\sum_{t=0}^{\infty} \mu^t (1-q)^t V = \frac{1}{1-\mu(1-q)} V
\]  

(12.24)

Thus, the discount factor \( \delta \) can be reinterpreted as follows:

\[
\delta = \mu (1-q)
\]  

(12.25)

The relationship between the discount factor and the discount rate is given by:

\[
\delta = \frac{1}{1+r}, \mu = \frac{1}{1+s}
\]  

(12.26)

Inserting (12.26) into (12.25) yields:

\[
\frac{1}{1+r} = \frac{1}{1+s} (1-q)
\]  

(12.27)

Solving for \( r \) and rearranging gives the reinterpretation for \( r^{380} \):

\[
r = \frac{s+q}{1-q}
\]  

(12.27)′

Therefore:

**Proposition 30:** The strong assumption of indefinite repetition can be relaxed by assuming uncertainty with respect to the game’s conclusion.

It seems plausible that this is very often the case in business relations, where parties can express their judgement of the relationship continuing rather in terms of probabilities than in terms of definite dates of conclusion\(^{381}\).

\(^{380}\) e.g. see Gibbons (2001), p. 9 footnote

\(^{381}\) Still, experiments suggest that parties will often find some way of cooperating in games with finite repetition, contrary to the logic of the presented argument. Having mentioned this behavioural evidence for something like a “trust mechanism” it will be ignored in the following as this thesis stands firmly on the grounds of rational decision making and opportunistic behaviour in its formal part. Still, as the reinterpretation of the rational model for uncertain ending shows, ignoring behavioural ideas does not come at such high a price as is often suggested.
Another advantage of this reinterpretation is that it actually enriches the model with a further variable. It is not only the discount rate (sometimes very aptly called “patience” rate) of one party whose reputation is at stake, which is relevant, but also the probability of the relationship's continuity, which is a judgement involving the relationship and thus both parties.

5.3.5 A Multiparty Extension

A common multiparty interpretation for the models presented is to assume that, in the relationship between an employer and his workers, workers live for one period but pass their experience on to fellow workers, who will in turn act in the following period as if the experience of their colleagues were their own\(^\text{382}\). This can be easily extended to the case where the workers live more than one period, but are spreading the news to their fellow workers. This can be modelled by adding a growth rate in the spirit of Section 5.3.2., reinforcing the reputation effect.

This is only one step short of assuming that there is a market reputation effect. The only difference, indeed, is that it is implicitly assumed in the case of the workers that the spreading of news is in some way facilitated by the fact that they work within the same organization. This highlights the importance of some kind of news-spreading mechanism.

It can very well be imagined that e.g. in the consultant-client relationship, observed shirking or reneging can play a role beyond the original relationship. If the project is highly visible, public interest will make sure that judgements are spread, possibly by press coverage. If the client or the consultant is very well entrenched in business circles, they will also have plenty of opportunity to spread around their experience. The mechanisms in place require close scrutiny which is probably more a sociological task. From an economic perspective, it can be asked, what motivates parties to spread news. The threat to do so can help in contractual relationships, but the same threat can be used for blackmailing. Therefore, the credibility of such comments is questionable.

Yet, the point is an important one. Consulting companies insist that their business is done very much on recommendation. Therefore, the argument goes, concerns for reputation make bonus contracts redundant. It does not, however, seem plausible that this argument is true to the same extent for all cases. There

\(^{382}\) see e.g. Gibbons (2001), p. 9 footnote 6
clearly seems to be a difference if the client is a large multinational company or a small start-up, if the consultant is a small one person firm or a big international consultancy, if the project is highly visible or not.

5.3.6 Discussion

First best can be achieved if utilities are not discounted in an infinitely repeated version of the basic one period model. The intuition behind this argument is that players start by cooperating, but if one party starts to defect, they will defect forever after. If this is accepted to be the strategy of the players, the dominant strategy is to cooperate. The immediate gain from defecting is always overcompensated by the loss in later periods.

The main criticism is that infinite repetition and no discounting are very unrealistic assumptions. This argument breaks down as soon as there is an end to the game. Then, the dominant strategy in the last period will be to defect. The game will unravel backwards. If games are finite, first best cannot be achieved because of backward unravelling.

If there is a discount rate and conclusion of the game is uncertain, first best can only be sustained in special cases. In some cases there will also be a second-best solution. As a general rule, the reputation effects are more likely to sustain contracts if the discount rate of the party whose reputation is at stake is low, if the judgement of this party attributes a low probability to the scenario that the relationship is discontinued, and if the gains of relational trade are high and possibly expected to rise.

The two models discussed above can be seen as complementary: If effort is not contractible, but can be observed, reputation concerns of the agent can support flat fee contracts based on effort. If this is not possible, parties may switch to output-based bonus contracts; but these contracts may not be feasible if output is just observable and not contractible. Reputation concerns of the principal may solve this problem. This suggests a sequence of analysis: Only if agent reputation effects are too low or observability limited will bonus contracts be considered.

The reputation mechanism may work beyond the original bilateral relationship. A “news-spreading mechanism” has to be assumed in these cases. Often it is not explicitly modelled. The relevant variables will be project visibility and the parties’ position and credibility within the relevant community.

---

383 see Radner (1981)
5.4 Career Concerns - Learning

5.4.1 Introduction

So far, two different kinds of arguments have been presented: First of all, it was shown that time can help to write explicit multi-period contracts which can reduce imperfect risk-sharing compared to one-period contracts.

Then, it was argued in the theory of supergames that reputation concerns might allow parties to contract on contingencies which are observable but not verifiable. Although an explicit contract would not be enforceable, an implicit contract ties observed effort to future pay-offs. Pay-offs will be lower if one party defects because the counterparty stops trusting, which reduces the future gains of trade. Thus, parties may be able to eschew switching to alternative performance measures which induce imperfect risk-sharing or are more distorted.

Another such implicit contract which ties current effort to future pay-offs is described by the model of career concerns. The intuition, as formulated by Fama\(^{384}\), is that there is no need for explicit contracts, because the market can effectively police agents. This is because the agent’s career will depend on his performance track record. The market monitors past performance and only agents who achieve high performance levels will be promoted. So, they will exert effort in order to positively affect their career chances.

This intuition is formalized by Holmström\(^{385}\). He shows that Fama’s conclusion that career concerns can provide efficient incentives will only hold under very special assumptions. Major inefficiencies can arise both in the short and in the long run.

The model allows important insights into incentive effects in a setting where the market monitors the agent’s output in order to learn about his productive capabilities. The following propositions will be derived:

31. It can be seen that incentives are high if the discount rate is low, if the precision of the production technology is high, and if the disutility of effort does not increase too fast.

\(^{384}\) Fama (1980)

\(^{385}\) Holmström (1982 reprinted in 1999) model will be presented in the following Section. His reasoning will be somewhat adjusted to make it easier for the non-technical reader to appreciate the argument. Minor errors of the article are corrected.
32. Interpreting the level of incentives in the stationary state, it can be said that incentives will never be higher than the efficient level. They will always be efficient if the discount rate is zero (\(\beta = 1\)). This was Fama's result. The result requires that there is some noise in the competence process (however small). \(\mu^* < 1 \Rightarrow \tau > 0 \Rightarrow \sigma^2_\mu > 0\). As soon as the discount rate is different from zero, incentives will be lower than the efficient level. Incentives will be closer to efficiency if the discount rate is low, updating of beliefs is fast and utility of effort increases slowly.

33. If precision of beliefs is initially lower than in the stationary state, speed of updating is high and therefore incentives are high. As they approach the stationary state over time precision increases, speed of updating decreases and incentives become lower. The opposite holds true if the precision of beliefs is initially higher than in the stationary state. In this case, incentives are low in the beginning and become higher over time. Therefore, the system is stable.

5.4.2 The Basic Model

It is assumed in the model of career concerns that the agent’s output in each period \(y_t\) depends on his effort \(a_t\), his productive capability \(\eta\) and a sequence of unrelated shocks \(\varepsilon_t\) representing the external factor:

\[
y_t = \eta + a_t + \varepsilon_t, \quad t = 1, 2, ... \tag{12.28}
\]

where \(\varepsilon_t\) is normally distributed with 0 mean and a variance of \(\sigma^2_\varepsilon\).

\[
\varepsilon_t \sim N(0, \sigma^2_\varepsilon) \tag{12.29}
\]

It is further assumed that both the agent and the principal do not know the agent’s productive capability. They do, however, share the same prior beliefs. These beliefs are represented by an initial assessment \(m_0\) of the agent’s capabilities and the assumed precision of these beliefs \(h_0\) (which equals the inverse of the variance \(h_0 = 1/\sigma^2_\varepsilon\)). As time proceeds, these beliefs are updated on the basis of the agent’s performance track record. In the models of signaling and screening it is assumed that the agents have private information on their own capability. In these cases it will be explored if it is possible to extract information from the agents. In the model of career concerns, however, it can be seen that information is assumed to be imperfect but symmetric: The agent and the market monitor the same normal learning process.
The focus here will be to show that, in the described setting, career concerns will create incentives in the absence of explicit incentive contracts. This is done by creating an indirect link between current effort and future compensation. The objective of this model set up by Holmström is to formalize the well-known argument of Fama, who went so far as to claim that career concerns will make explicit incentive contracts redundant by providing efficient incentives costlessly. It is therefore central to understand why the agent should believe that his current effort would positively effect his future compensation.

First, the agent’s problem is considered. His pay-off in any period \( t \) equals his compensation \( c_t \) minus his disutility of effort \( g_t(a_t) \), which is assumed to be an increasing and convex function in effort.

\[
u_t = c_t - g_t(a_t)
\]  

where

\[
g_t(a_t) > 0, g_t'(\cdot) > 0, g_t''(\cdot) > 0 \tag{12.31}
\]

The agent is not only concerned about his current pay-off, but tries to choose effort in order to maximize the present value of current and future pay-offs. It is obvious that this present value does not only depend on the current choice of effort, but also on all choices of effort in the future; but future decisions cannot be made today, because the information on which they are based is future information and therefore not currently available. Therefore, the agent’s problem is to solve for the optimal decision rule which prescribes in every period \( t \) which decision \( a_t \) will be taken contingent on the basis of the information \( y_{t-1} \), which will then be available. This will automatically produce the optimal current choice of effort by setting in current information:

\[
a_t = a_t(y_{t-1}) \tag{12.32}
\]

The fact that future information is not available today also implies that the agent is faced with a decision under uncertainty. It is assumed that the agent is risk-neutral and therefore maximizing expected present value. The optimal decision rule is therefore the solution to the following maximization problem:

\[
a^*() \in \arg \max_{a^*() \in a^*(\cdot)} \sum_{t=1}^{\infty} \beta^{t-1} \left[ E c_t - E g_t\left(a_t\left(y_{t-1}\right)\right) \right] \tag{12.33}
\]
where $\beta$ is the discount factor and $a^*(\cdot)$ is a vector representing the optimal decision rule.

It will now be analysed, what determines compensation. It is assumed that the agent faces a competitive risk-neutral market. This implies that his compensation equals expected marginal output\(^{386}\):

$$c_t = E(y_t)$$  \hspace{1cm} (12.34)

It follows from the production function (12.28) that the market determines compensation by adding expected capability and expected choice of effort.

$$c_t = E_t(\eta) + E(a_t)$$  \hspace{1cm} (12.35)

It is obvious from the production function that effort is a substitute for capability. Therefore, the whole game played by the agent is to choose effort in order to bias the learning process of the market in his favour. But this is anticipated by the market. One might think that, because his action cannot be observed, asymmetric information will develop over time, but this is not the case\(^{387}\): His maximizing behaviour makes him perfectly predictable if his utility function is assumed to be common knowledge, in line with the usual assumptions of agency theory. His is trapped. He cannot fool the market, but if he did not show maximizing behaviour he would bias the learning process against him\(^{388}\). Therefore, expected effort choice in (12.35) equals the agent's optimal decision rule:

$$E(a_t) = a_t^* (y_{t-1})$$  \hspace{1cm} (12.36)

---

386 Note that the assumption of competitive markets is a short-cut for saying that the agent faces a number of principals and that there is competition among these principals, driving their profits down to zero. This assumption is not as strong as it seems. Indeed, the market model could be replaced by a bargaining model, which would make the argument more complex but would not change its insights. So, the zero-profit hypothesis is just a way of holding one party's utility constant while maximizing the utility of the other party, which insures Pareto optimality. This is in the same spirit as setting agents' utility to their reservation level as was done in other instances.

387 see Gibbons (2001)

388 Holmström (1999) calls this situation a "rat race".
It was also mentioned that the market assesses capability on the basis of the agent’s performance track record. So, assessment \( m_t \) in period \( t \) is a function of the assessment at the beginning of the last period \( t-1 \), updated by the observed outcome at the end of the last period \( y_{t-1} \). As

\[
y_{t-1} = \eta + a_{t-1} + \epsilon_{t-1} \tag{12.37}
\]

and \( a_{t-1} \) can perfectly be anticipated (\( a_{t-1} = a^*_{t-1} \)), (12.37) can be written as

\[
z_{t-1} = y_{t-1} - a_{t-1} = \eta + \epsilon_{t-1} \tag{12.38}
\]

where \( z_t \) is a sequence of the agent’s capability disturbed by an error term. Updating the market’s beliefs on the agent’s capability then occurs by calculating the weighted average of the initial belief and the observation. The weights are the precision of the initial belief \( h_{t-1} \) and the precision of the observation \( h_e \), respectively.

\[
E_t(\eta) = m_t = \frac{h_{t-1}m_{t-1} + h_e z_{t-1}}{h_{t-1} + h_e} = \frac{h_t m_t}{h_t (1 - t)}h_e \tag{12.39}
\]

The market’s assessment of the agent’s capability becomes ever more precise as the learning process continues (precision is increasing in \( t \)):

\[
h_t = h_{t-1} + h_e = h_t + (t - 1)h_e \tag{12.40}
\]

As the agent’s maximization problem for finding the optimal decision rule depends on compensation (see (12.33)) but compensation in turn depends on the optimal decision rule (see (12.35) and (12.36)), there is an interdependence between the two decision problems, which means that they have to be solved simultaneously. Rewriting (12.33) and inserting (12.39) and (12.36) into (12.35), this simultaneous decision problem can be stated as:

\[
a^* (\cdot) \in \arg \max_{a(\cdot)} \sum_{t=1}^{\infty} \beta^{t-1} \left[ E_t - Eg_t \left( a_t \left( y_{t-1} \right) \right) \right] \tag{12.33}
\]
And:

\[ c_t = \frac{h_t m_t + h_t \sum_{s=1}^{t-1} z_s}{h_t} + a_t^* (y_{t-1}) \]  

(12.35)

This is solved by taking expectation of (12.35)\(^{389}\)

\[ E_c_t = \frac{h_t m_t}{h_t} + h_t \sum_{s=1}^{t-1} \left[ m_s + a_s - E a_s^* (y_{s-1}) \right] + E a_t^* (y_{t-1}) \]  

(12.35)′′

and inserting the resulting (12.35)′′ into (12.33). Then, the first order conditions \( y_t, t = 1, 2, \ldots \) can be written as\(^{390}\):

\[ \gamma_t = \sum_{s=t+1}^{\infty} \beta^{s-t} \frac{h_s}{h_t} = g'(a_t). \]  

(12.41)

**Proposition 31:** It can be seen that incentives are high if the discount rate is low\(^{391}\), if the precision of the production technology is high\(^{392}\) and if the disutility of effort does not increase too fast.

Efficient incentives are characterized by a situation where marginal product equals marginal cost:

\[ g'(a) = 1 \]  

(12.42)

In the model described so far, equilibrium will be very inefficient: In the long run (\( t \to \infty \)) it can be seen that there will be no incentives from career concerns (\( \gamma_t \to 0 \)) as the assessment will become indefinitely precise (\( h_t \to \infty \)). This is an intuitive result: The agent will only have incentives to exert effort from career concerns, as long as his capability is not fully known.

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389 See Mathematical Appendix at the end of this Paragraph.
390 See Mathematical Appendix at the end of this Paragraph.
391 \( r \downarrow \Rightarrow \beta \uparrow \Rightarrow g'(a) \uparrow \Rightarrow a \uparrow \)
392 \( h_r \uparrow \Rightarrow g'(a) \uparrow \Rightarrow a \uparrow \)
**Mathematical Appendix:**

Footnote 389:

\[ Ec_t = \frac{h_m}{h_t} + E\left( E_{z_s} \sum_{s=1}^{h_s} z_s \right) + E_a^*(y_{t-1}) \]

\[ = \frac{h_m}{h_t} + h_s \sum_{s=1}^{h_s} E_{z_s} + E_a^*(y_{t-1}) \]  \hspace{1cm} (\alpha) \]

\[ z_s \text{ can be written as:} \]

\[ z_s = y_s - a_s = \eta + \varepsilon_s - a_s + a_s = \eta + a_s + \varepsilon_s - a_s \]  \hspace{1cm} (\beta) \]

Inserting (\beta) into (\alpha) gives:

\[ Ec_t = \frac{h_m}{h_t} + h_s \sum_{s=1}^{h_s} [m_s + a_s - E_a^*(y_{s-1})] + E_a^*(y_{t-1}) \]

Footnote 390:

\[ \frac{\partial}{\partial a_t} \sum_{s=t}^{S} \beta^{s-t} \left[ \frac{h_m}{h_s} + h_s \sum_{i=1}^{h_i} [m_i + a_i - E_a^*(y_{i-1})] + E_a^*(y_{s-1}) - E_{g_s}(a_s^*(y_{s-1})) \right] \]

\[ = \sum_{s=t}^{S} \beta^{s-t} E_a^*(y_{s-1}) - \sum_{s=t}^{S} \beta^{s-t} E_{g_s}(a_s^*(y_{s-1})) \]

\[ = 0 + \sum_{s=t+1}^{S} \beta^{s-t} \frac{h_{s-1}}{h_s} + 0 - g'(a_t) = \sum_{s=t+1}^{S} \beta^{s-t} \frac{h_{s-1}}{h_s} - g'(a_t) = 0 \]

**5.4.3 Extension: Adding Innovation**

The situation changes if a plausible assumption is added to the basic model. This is by assuming that competence is not invariant over time but is modelled as an...
autoregressive process. More specifically, it is assumed that this period's capability equals last period's capability plus a stochastic shock:

\[ \eta_t = \eta_{t-1} + \delta_{t-1} \]  \hspace{1cm} (12.43)

where the sequence of stochastic shocks is driftless with variance \( \sigma_\delta^2 \):

\[ \delta_{t-1} \sim N(0, \sigma_\delta^2) \]  \hspace{1cm} (12.44)

The shocks can be interpreted as reflecting innovation. Innovation changes job characteristics. Therefore, someone who was well qualified to do the job in the past is not necessarily well qualified to do the job now or in the future.

Having motivated the noise term in the competence process, it will now be analysed which effects this assumption has on incentives. In fact, beliefs are still updated by calculating the weighted average of last period's initial beliefs and last period's observation, with the precision of the initial beliefs and the precision of the observation as weights, respectively:

\[ m_t = \mu_{t-1} m_{t-1} + (1 - \mu_{t-1}) z_{t-1} \]  \hspace{1cm} (12.45)

where

\[ \mu_{t-1} = \frac{h_{t-1}}{h_{t-1} + h_\epsilon} \]  \hspace{1cm} (12.46)

What changes is the way the precision of the belief is assessed. By updating the initial belief it will be made more precise.

\[ \hat{h} = h_{t-1} + h_\epsilon \]  \hspace{1cm} (12.47)

But the belief refers to last period's capability, which is irrelevant because last period's decisions have already been made before updating occurs. What is interesting is the current period's assessed capability. Contrary to the basic model, it is now assumed that last period's capability does not fully determine present capability. It is still the best estimate, which is why beliefs are updated in the same way. Yet, the precision of the present belief about capability will be lower than \( \hat{h} \) because innovation adds uncertainty of whether someone who was capable in the past will also be capable at present and in the future.
\[
\frac{1}{h_t} = \frac{1}{h_{t-1} + h_e} + \frac{1}{h_s} = \frac{h_s + h_{t-1} + h_e}{(h_{t-1} + h_e)h_s}
\]  
(12.48)

What happens is that first precision is increased by making another observation and then decreased by adding the noise \(\left(\sigma_s^2 = 1/h_s\right)\) of the competence process. Obviously, one can try to solve for a stationary state which is defined as the state where decrease of noise due to learning is exactly offset by the increase of noise due to innovation. It will later be shown that the stationary state actually is a stable equilibrium.

For technical reasons, the stationary state is calculated in terms of \(\mu_s\), which are tied to \(h_s\) by expression (12.46). It can be shown that:\(^{393}\)

\[
\mu_t = \frac{1}{2 + r - \mu_{t-1}}
\]
(12.49)

In the stationary state the precision does not change anymore from one period to the other:

\[
h^* = h_t = h_{t-1} \Rightarrow \mu^* = \mu_t = \mu_{t-1}
\]
(12.50)

Inserting (12.50) into (12.49) gives:

\[
\mu^* = \frac{1}{2 + r - \mu^*} \Leftrightarrow -\mu^* + (2 + r)\mu^* - 1 = 0
\]
(12.51)

Solving the quadratic equation gives:

\[
\mu_{1/2}^* = 1 + \frac{1}{2}r \pm \sqrt{\frac{1}{4}r^2 + r}
\]
(12.52)

From (12.45) follows that \(\mu^* < 1\). Therefore:

\[
\mu^* = 1 + \frac{1}{2}r - \sqrt{\frac{1}{4}r^2 + r}
\]
(12.53)

---

393 See Mathematical Appendix at the end of this Paragraph.
Taking account of the modified learning process:

\[ m_t = m_1 \prod_{i=1}^{1-l} \mu_i + \sum_{s=1}^{1-l} z_s \left( \prod_{i=s+1}^{1-l} \mu_i \right) (1-\mu_s) \]  

(12.54)

the market’s compensation rule can once again be inserted into the agent’s maximization problem. The first order conditions \( \gamma_t \) can be written as\textsuperscript{394}:

\[ \gamma_t = (1-\mu_t) \sum_{s=t+1}^{\infty} \beta^{s-1} \prod_{i=t+1}^{s-1} \mu_i = g'(a_t) \]  

(12.55)

In the stationary state, first-order conditions can be further simplified\textsuperscript{395}:

\[ \gamma_t \equiv \frac{(1-\mu^*) \beta}{1-\beta \mu^*} = g'(a_t) \]  

(12.56)

**Proposition 32**: Interpreting the level of incentives in the stationary state, it can be said, that incentives will never be higher than the efficient level:

\[ \frac{\beta (1-\mu^*)}{1-\mu^* \beta} = g'(a^*) \leq 1 \]  

(12.57)

They will always be efficient if the discount rate is zero \( (\beta = 1) \). This was Fama’s result. The result requires that there is some noise in the competence process (however small). \( \mu^* < 1 \Rightarrow \tau > 0 \Rightarrow \sigma_{z}^2 > 0 \).

\[ \frac{1-\mu^*}{1-\mu^*} = 1 \]  

(12.58)

As soon as the discount rate is different from zero, incentives will be lower than the efficient level. Incentives will be closer to efficiency if the discount rate is low, updating of beliefs is fast and utility of effort increases slowly.

\textsuperscript{394} See Mathematical Appendix at the end of this Paragraph.

\textsuperscript{395} See Mathematical Appendix at the end of this Paragraph.
Updating is a very intuitive concept. It is the weight \((1 - \mu^*)\) which is given to the most recent observation. Updating will be fast if \(\mu^*\) is low, which happens if the precision of the production process is high relative to the precision of the competence process, or equivalently if the competence process is very disturbed relative to the production process. These results are recorded in Exhibit 10.

Exhibit 10: Career Concerns: Incentives in Equilibrium

The level of incentives is shown as a function of the speed of updating for different discount rates\(^{396}\). The surprising result is that Fama's prediction of efficient incentives becomes true for zero discount rate even at an infinitesimally small updating speed (if only a small amount of noise is added to the competence process), as was mentioned before.

---

\(^{396}\) In this Exhibit, \(r\) represents the discount rate (not to be confused with the \(r\) in the text which refers to relative precision of the output process compared to the competence process).
Mathematical Appendix:

Footnote 393:

\[
\mu_i = \frac{h_i}{h_i + h_\varepsilon} = \left(1 + \frac{h_\varepsilon}{h_i}\right)^{-1} \quad \text{(a)}
\]

Inserting (12.48) into (a) gives:

\[
\mu_i = \left(1 + \frac{h_\varepsilon}{h_\delta} \left(\frac{h_\varepsilon + h_{i-1}}{h_\delta + h_\varepsilon}\right)^{-1}\right)^{-1} = \left[1 + \frac{h_\varepsilon}{h_\delta} \left(\frac{h_{i-1}}{h_\delta + h_\varepsilon} + 1\right)\right]^{-1} \quad \text{(b)}
\]

Solving (12.46) for \(h_{i-1}\) gives:

\[
h_{i-1} = \frac{h_\varepsilon \mu_i}{1 - \mu_i} \quad \text{(\(\gamma\))}
\]

Inserting (\(\gamma\)) into (b) gives:

\[
\mu_i = \left\{1 + \frac{h_\varepsilon}{h_\delta} \left[\left(\frac{h_\varepsilon + h_{i-1}}{h_\delta 1 - \mu_i}\right)^{-1} + 1\right]\right\}^{-1}
\]

Setting \(r = \frac{h_\varepsilon}{h_\delta}\) gives:

\[
\mu_i = \left\{1 + r \left[\left(r^{-1} \left(1 + \frac{\mu_i}{1 - \mu_i}\right)^{-1} + 1\right)\right]\right\}^{-1} = \left\{1 + 1 - \mu_{i-1} + r\right\}^{-1}
\]

\[
= \frac{1}{2 + r - \mu_{i-1}}
\]
Footnote 394:

\[
E_{c_t} = m_t \prod_{i=1}^{s-1} \mu_i + \sum_{s=1}^{l-1} \mathbb{E}(z_s) \prod_{i=s+1}^{l-1} \mu_i (1-\mu_s) + E_{a_t^*}(\mu_{t-1})
\]

Setting \( z_s = \eta + a_s + \varepsilon_s - a_s \), it can be written:

\[
E_{c_t} = m_t \prod_{i=1}^{s-1} \mu_i + \sum_{s=1}^{l-1} \left\{ (m_t + a_s - E_{a_s^*}(y_{s-1})) \left[ \prod_{i=s+1}^{l-1} \mu_i \right] (1-\mu_s) \right\} + E_{a_t^*}(y_{t-1})
\]

Inserting in the agent’s maximisation problem and taking the derivative gives:

\[
\frac{\partial}{\partial a_t} \sum_{s=1}^{\infty} \beta^{s-t} \left[ E_{c_s} - E_{g_s}(a_s(y_{s-1})) \right]
\]

\[
= \left\{ \frac{\partial}{\partial a_t} \sum_{s=1}^{\infty} \beta^{s-t} m_t \prod_{i=1}^{s-1} \mu_i \right\} + \left\{ \frac{\partial}{\partial a_t} \sum_{s=1}^{\infty} \beta^{s-t} \sum_{i=1}^{s-1} \left( m_t + a_i - E_{a_i^*} \left[ \prod_{j=i+1}^{s-1} \mu_j \right] (1-\mu_i) \right) \right\}
\]

\[
+ \left\{ \frac{\partial}{\partial a_t} \sum_{s=1}^{\infty} \beta^{s-t} E_{a_t^*} \right\} - \left\{ \frac{\partial}{\partial a_t} \sum_{s=1}^{\infty} \beta^{s-t} E_{g_s}(a_s) \right\}
\]

\[
= 0 + \sum_{s=t+1}^{\infty} \beta^{s-t} \cdot \prod_{j=t+1}^{s-1} \mu_j (1-\mu_j) + 0 - g'(a_t)
\]

Therefore, the first order condition \( \gamma_t \) can be written as:

\[
\gamma_t = (1-\mu_t) \sum_{s=t+1}^{\infty} \beta^{s-t} \prod_{i=t+1}^{s-1} \mu_i = g'(a_t)
\]

(Readers comparing this result with the original Holmström article will note that there is a typing error in the original.)
Applying the formula of the sum of infinite geometric sequences to the right hand side gives:

\[ \gamma_i = (1 - \mu^*) \sum_{i=1}^{\infty} \beta^i \mu^{i-1} = g'(a_i) \]

\[ \gamma_i = (1 - \mu^*) \frac{1}{\mu} \left( \frac{1}{1 - \beta \mu^*} - 1 \right) = g'(a_i) = (1 - \mu^*) \frac{1}{\mu} \left( \frac{\beta \mu^*}{1 - \beta \mu^*} \right) = g'(a_i) \]

\[ \gamma_i = \frac{(1 - \mu^*) \beta}{1 - \beta \mu^*} = g'(a_i) \]

The trick is that, in the stationary state, \( \mu_i \)'s with \( i \geq \bar{t} \) equal \( \mu^* \). Also note the convention that

\[ \prod_{s=1}^{s-1} \mu_i = 1. \]

5.4.4 Disequilibrium – Transient Effects

It is important to understand what happens before the stationary state is reached. This will be particularly relevant in situations where updating is slow and periods are very long. In such a situation, not many “learning loops” are possible in a given span of time, the observed signal is bad and competence is very stable over time. So, the market tends to stick with its prior beliefs. Here, the stationary state might never be reached in the considered period and disequilibrium might be the only relevant state.

The dynamics of incentives are studied by doing comparative statics in the first order conditions (12.55) of the agent's maximization problem. It can be seen that incentives increase in all periods if updating speed increases \( (\mu_i \text{ decreases})^{397} \). The weight \( \mu_i \) is an increasing function of \( h_i \) as can be seen in (12.46). Therefore, as precision of beliefs with respect to capability \( (h_i) \) increases, updating becomes slower and incentives decrease.

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397 see Holmström (1999), p. 174 for the formal proof by induction
As $\mu_t$ is an increasing function of $\mu_{t-1}$ as can be seen in (12.49), and there is only one stationary state in the relevant interval $(0,1)$, it can be seen from Exhibit 11\(^{398}\) that if last period's speed of updating was higher than in the stationary state ($\mu_{t-1} < \mu^*$) – which is equivalent to saying that the precision of last period's belief was lower than the precision of beliefs in the stationary state ($h_{t-1} < h^*$), the speed of updating will decrease ($\mu_t > \mu_{t-1}$) or the precision of beliefs will increase ($h_t > h_{t-1}$). This means that the sequence $\mu_t$ will be approaching $\mu^*$ from below.

\[
\mu_t = \frac{1}{2 + r - \mu_{t-1}}
\]

Exhibit 11: Career Concerns: Incentives in Disequilibrium

It can therefore be said that

**Proposition 33:** If precision of beliefs is initially lower than in the stationary state, speed of updating is high and therefore incentives are high. As they approach the stationary state over time precision increases, speed of updating decreases and incentives become lower. The opposite holds true if the precision of beliefs is initially higher than in the stationary state. In this case,

\(^{398}\) Taken with small modification from Holmström (1999), p. 175
Incentives are low in the beginning and become higher over time. Therefore, the system is stable.

5.4.5 Discussion

Holmström's formalization of Fama's argument shows that, in equilibrium, career concerns will provide efficient incentives only under very special conditions. The most notable assumption is a zero-discount rate. There also has to be some change of job characteristics due to innovation in order to prevent the market from eventually fully learning the agent's capability.

If the discount rate is not zero, incentives will never be efficient in equilibrium. They will, however, be close to efficiency if the discount rate is low, job characteristics change considerably due to innovation, and the external factor in the output process is unimportant. In absolute terms, incentives will also be higher if the disutility of effort rises only slowly. A useful and intuitive concept in this context is the speed of updating. In fact, if the noise of the production function is low relative to the noise of the competence process, the speed of updating will be high, which in turn will make incentive increase since the agent knows that higher effort will have an impact. If, however, talent will be expected to last forever, once it has been proven and potential signals to the contrary are very unreliable, one will stick to the prior belief.

Therefore, if the discount rate is low and updating fast, there will be fairly high incentives to perform in equilibrium. The intuitive idea is that reputation is not worth so much, or high reputation has to be reproven very often. In this situation, career concerns solve the shirking problem. Conversely, if the discount rate is high (the agent does not care about the future) and updating is slow, there will be low incentives in the steady state.

In some situations, incentives in disequilibrium will be very important. This is especially true in cases where updating is slow and the frequency of observation is low (due to a long production process). Incentives will be initially high if the precision of the initial belief is thought to be higher in later periods. Incentives will be initially low if the precision of the initial belief is thought to be lower in later periods. The first case will be true in most cases. In the beginning, not much is known about the agent. Therefore, beliefs will be relatively imprecise.

In a situation where the noise of the production function is high relative to noise of the competence process, which is equivalent to a situation where there is little change of job characteristics due to innovations, and observation of output is only a very bad signal due to an important external factor, it was said that
updating will be slow and incentives will be much too low in equilibrium. If there is low initial precision it was also said that incentives will be much too high in the beginning. This situation will persist for quite some time as the speed of updating is low. Therefore, an agent working in an industry where job characteristics are not expected to change much and the external factor is very important, as might be expected for many service industries, will get very inefficient incentives from career concerns.