Appendix A

Tobin’s q-Theory of Investment

Tobin’s q is defined as the ratio of the real market value of physical capital goods $MV_r$ to the real reproduction or real replacement costs of those goods $V_r$, being equal to the ratio of the nominal market value of physical goods $MV = MV_r P$ to the nominal value of replacement costs of those goods $V = V_r P$, where $P$ denotes the price level. Formally, Tobin’s q is given as

$$q = \frac{MV_r}{V_r} = \frac{MV}{V}.$$

In case the market value of existing capital goods, which are traded e.g. in the stock exchange, is larger than replacement costs, i.e. if it holds that $q > 1$, then the rate of investment (Tobin 1969, p. 143), denoting the speed at which investors want to increase the capital stock, is positive, since buying new capital goods at replacement costs is cheaper than buying already existing capital goods at the stock exchange. By way of contrast, if the market value of capital goods is smaller than replacement costs, i.e. in case it holds that $q < 1$, the rate of investment is negative, because buying already existing capital goods at the stock exchange is cheaper than buying new capital goods at replacement costs. As a result, according to Tobin’s q-theory of investment, the investment function, expressing the “desired” or “demanded” rate of investment $I/K = \dot{K}/K = \dot{K}$, being equal to the growth rate of the capital stock, where $I$ denotes real investment and $K$ the real capital stock, is positively dependent on $q$. Hence, one possible form of a $q$-investment function could be expressed formally as

$$\frac{I}{K}(q) = \pi (q - 1),$$

where $\pi > 0$ is a positive constant. Thus, if $q < 1$ then $I/K < 0$, if $q > 1$ then $I/K > 0$, and if $q = 1$ then $I/K = 0$. This positive correlation between the rate of aggregate investment and the market value/replacement cost ratio has been also emphasized by Keynes who wrote,

... The daily revaluations of the Stock Exchange, though they are primarily made to facilitate transfers of old investments between one individual and another, inevitably exert a decisive influence on the rate of current investment. For there is no sense in building up a new enterprise at a cost greater than that at which a similar existing enterprise can be purchased; whilst there is an inducement to spend on a new project what may seem an extravagant sum, if
it can be floated off on the Stock Exchange at an immediate profit (Keynes, 1936 (1936), p. 151).

In order to specify a q-investment function, variables determining Tobin’s q, i.e. variables determining the real market value \( MV_r \) and real replacement costs \( V_r \) of physical capital assets, have to be identified. The real market value of capital goods at time \( t \), \( MV_r(t) \), is determined by discounting expected real earnings at future times \( s \geq t \), \( X^e_r(s) \), by a real discount factor \( z_k \) representing investors’ “required” real rate of return for holding physical assets, i.e. formally it holds that

\[
MV_r(t) = \int_{t}^{\infty} X^e_r(s) e^{-z_K(s-t)} ds.
\]

The discount rate \( z_K \) is not the long-term government bond rate, or the interest rate on long-term corporate bonds, but a rate which is appropriate for the valuation of future earnings, containing the risk and uncertainties of business firms’ cash flows. Real replacement or real reproduction costs of physical capital assets at time \( t \), \( V_r(t) \), are determined by discounting expected real earnings at times \( s \geq t \), \( X^e_r(s) \), by the marginal efficiency of capital \( Z \) (internal rate of return, or real profit rate on physical assets), i.e. formally it holds that

\[
V_r(t) = \int_{t}^{\infty} X^e_r(s) e^{-Z(s-t)} ds.
\]

Assuming a constant value of expected real earnings for all periods \( s \geq t \), i.e. \( X^e_r(s) = X^e \), the real market value, real replacement costs and Tobin’s q, all at time \( t \), are defined as

\[
MV_r(t) = \frac{X^e}{z_K}, \quad V_r(t) = \frac{X^e}{Z}, \quad q(t) = \frac{MV_r(t)}{V_r(t)} = \frac{X^e}{X^e_rZ} = \frac{Z}{z_K}.
\]

As a result, Tobin’s q, and therefore the rate of aggregate investment is positively dependent on the ratio of the current profit rate to the risk-adjusted discount rate investors require to hold physical assets.

In order to obtain a relationship between the rate of return relation \( Z/z_K \), and the relation of stock prices to the general price level, Tobin’s q can be alternatively determined by defining the nominal market value at time \( t \), \( MV(t) \), of existing capital goods \( K \) as

\[
MV(t) = P_K(t) K(t),
\]

where \( P_K(t) \) denotes the current market price of physical assets at time \( t \), i.e. the so-called demand price of capital, which can be approximated by actual stock prices. By way of contrast, new capital goods are valued at their nominal replacement costs, i.e. by the so-called supply price of capital being equal to the current price level \( P(t) \); as a result, the nominal value of new capital goods valued at replacement costs at time \( t \), \( V(t) \), is defined as

\[
V(t) = P(t) K(t).
\]
Consequently, Tobin’s $q$ can be alternatively expressed in nominal terms as

$$q(t) = \frac{MV(t)}{V(t)} = \frac{P_K(t)K(t)}{P(t)K(t)} = \frac{P_K(t)}{P(t)},$$

stating that $q$, and thereby the rate of investment, is positively influenced by the ratio of capital goods’ market prices (stock prices) to the general price level. Putting all definitions together and neglecting time $t$, Tobin’s $q$ is given by

$$q = \frac{MV_r}{V_r} = \frac{MV}{V} = \frac{Z}{z_K} = \frac{P_K}{P}.$$
