Chapter 2

The Basic Model

In this section we introduce the model. It is based on the model of LRTb, but for simplicity we will neglect the fixed costs (which does not change the results qualitatively). On the other hand, we extend the model by adding the call externality.

2.1 Cost and Price Structure

Imagine a telecommunications market which is served by two interconnected networks labeled 1 and 2. Both networks have full coverage, i.e. every consumer can be reached by all other consumers, no matter which network they are subscribed to. The marginal cost of originating or terminating a call is $c_0 > 0$, and the total marginal cost of a call is

$$c = 2c_0 + c_1,$$

where $c_1 \geq 0$ is the marginal cost of transmitting a call from the originating to the terminating end. The reciprocal unit access charge is $a \geq -(c_0 + c_1)$. Note that access charges may be negative. Setting a negative access charge corresponds to subsidising termination. The subsidy cannot be larger than the costs of originating and transmitting a call, however, since otherwise a network could make unlimited profits by installing a computer which permanently calls into the rival network.
Networks either compete in linear prices $p_{ii}$ (for on-net calls within network $i$) and $p_{ij}$ (for off-net calls originating in network $i$ and terminating in network $j$), or, in the case of two-part tariffs, also in the fixed fee $F_i$.

2.2 Subscription Decision and Demand

On the demand side there is a large number of consumers, formally a continuum. A consumer can be member of at most one network. From the consumers’ point of view the networks are horizontally differentiated, and this differentiation is of the Hotelling type. The networks are located at the extreme points of the unit interval $[0, 1]$, and each consumer is located at some address $x \in [0, 1]$. The total mass of consumers, normalized to 1, is distributed uniformly on this interval. The degree of horizontal differentiation is measured by a parameter $t$ corresponding to the “transport costs”. This means that a consumer located at address $x$ faces a disutility of $t|x - x_i|$ if he subscribes to network $i$, where $x_1 = 0$ and $x_2 = 1$ are the locations of the two networks.\(^1\)

Consumers have homogeneous preferences for calls to other consumers. Calls to distinct other consumers constitute independent goods and total utility is assumed to be additively separable. The utility from an active call of length $q$ is given by $u(q)$, where $u' > 0$ and $u'' < 0$. For technical reasons we also assume that the Inada conditions $\lim_{q \to 0} u'(q) = \infty$, $\lim_{q \to \infty} u'(q) = 0$ are fulfilled, guaranteeing strictly positive and finite demand for all positive prices. It is helpful here to imagine that each consumer makes exactly one call to each other consumer, and only the length of a call is variable.

Consumers also derive utility from receiving calls. We denote the passive utility of receiving a call of length $q$ by a strictly increasing and strictly concave function $\bar{u}(q)$ with the same qualitative properties as $u(q)$.

A consumer with income $y$, subscribed to network $i$ and located at $x$, making a call of length $q_{out}$ to some other consumer and receiving a call of length $q_{in}$ from some consumer, enjoys a total utility of

$$v_0 + y + u(q_{out}) + \bar{u}(q_{in}) - t|x - x_i|,$$

\(^1\)It is not necessary to take the disutility interpretation of the Hotelling transport costs literally, $t$ may simply be interpreted as a parameter measuring the intensity of price competition.
where $v_0$ is some fixed surplus from being connected, large enough to guarantee full participation, i.e. to prevent consumers from not subscribing in equilibrium.

The timing is as follows. First, networks cooperatively choose a reciprocal access charge, then they (noncooperatively) set on- and off-net prices, and the fixed fee in case of two-part tariffs. Consumers choose a network to subscribe to and then they choose the length of their on- and off-net calls.

Let

$$q(p) = \arg\max_q \{u(q) - pq\}$$

be the consumer's demand, writing $q_{ij}$ short for the demand for on- and off-net calls $q(p_{ij})$. Denoting by

$$v(p) = \max_q \{u(q) - pq\}$$

net surplus, under price discrimination with given market shares $\alpha_1$ and $\alpha_2$, network $i$ offers its subscribers a total net surplus of

$$w_i = \alpha_i[v(p_{ii}) + \bar{u}(q_{ii})] + \alpha_j[v(p_{ij}) + \bar{u}(q_{ji})] - F_i.$$  \hfill (2.1)

Letting

$$h_{ij} = v(p_{ij}) + \bar{u}(q_{ji}),$$

we may write

$$w_i = \alpha_i h_{ii} + \alpha_j h_{ij} - F_i.$$  \hfill (2.2)

### 2.3 Existence and Stability of Equilibria

#### 2.3.1 Existence of Consumer Equilibria

Imagine prices are fixed, and consumers have to decide which network to subscribe to. Due to the tariff-mediated network externalities and the call externality, a consumer's utility is influenced by all other consumers' subscription decisions. His decision problem is therefore not a simple optimization, but a strategic one. For fixed prices, consumers are actually playing a multi-person

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\[^2^\text{Throughout this work, let } \{i, j\} = \{1, 2\} \text{ if not indicated otherwise.}\]
game. Hence their subscription decisions will depend on their expectations about future market shares, i.e. on the subscription decisions of all other consumers. In this situation, the natural solution concept is the Nash equilibrium of the corresponding game. To distinguish between this game among consumers and the extended two-stage extensive-form game where networks decide on their prices prior to consumers' subscription decisions, we call a Nash equilibrium of the game among consumers a consumer equilibrium. Thus, a consumer equilibrium is given if the market shares are such that no consumer has an incentive to unilaterally deviate from his subscription decision and switch to the other network.

A consumer equilibrium need not be unique. To see this, suppose the on-net prices as well as the off-net prices are the same for both networks, but the on-net price is below the off-net price. Suppose also, that the degree of differentiation between networks is small. If all consumers expect all others to subscribe to network 1, then it is optimal for them also to subscribe to network 1. The same is true for network 2, however. In this case the game turns out to be a coordination game, where it is an equilibrium for consumers to coordinate on one of the networks. If this happens, then we say that the market is cornered by one network.

If the market is cornered, i.e. if \( \alpha_i = 1 \) for some \( i \), then even the consumer with the weakest preferences for network \( i \) (the consumer located at \( x_j \)) chooses to subscribe to this network. If the difference between on-net prices and off-net prices is not large enough to overcome the horizontal differentiation for this extreme consumer, the result of the subscription decisions cannot be a cornered market. Both networks will have a strictly positive market share in any consumer equilibrium then. Such an outcome is called a shared market equilibrium.

If there is a shared market equilibrium with \( 0 < \alpha_i < 1 \), then the consumer located at \( x = \alpha_1 \) is just indifferent between the networks. The market share \( \alpha_1 = \alpha \) (and \( \alpha_2 = 1 - \alpha \)) in a shared market equilibrium can thus be calculated from the indifference condition

\[
W_1 - t\alpha = W_2 - t(1 - \alpha),
\]

and reads

\[
\alpha = \frac{1}{2} + \sigma(W_1 - W_2). \tag{2.3}
\]
Here, \( \sigma = 1/2t \) measures the degree of substitutability between the two networks. If \( t \to 0 \), networks become perfect substitutes, whereas for \( t \to \infty \) substitutability vanishes and the networks become local monopolies.

Note that (2.3) gives the market shares only implicitly, since these occur also in \( w_i \) on the right-hand side. To calculate the market shares explicitly, we insert from (2.2), setting \( \alpha = \alpha_1 = 1 - \alpha_2 \). Solving for \( \alpha \) then yields

\[
\alpha = \frac{H_1}{H_1 + H_2},
\]

with

\[ H_i = 1/2 + \sigma(h_{ij} - h_{jj} + F_j - F_i). \]

Obviously, for a shared market equilibrium to exist, i.e. for \( \alpha > 0 \), \( H_1 \) and \( H_2 \) must have the same sign: \( H_1 H_2 > 0 \).

### 2.3.2 Stability of Consumer Equilibria

As already discussed above, there may be multiple consumer equilibria for given prices. However, some of these can usually be eliminated by pointing out that an economically meaningful equilibrium has to be stable with respect to an appropriate adjustment dynamic.

In the case where the consumers' game is a symmetric coordination game, there are two cornered market equilibria. However, there is also a third equilibrium, where each consumer subscribes to the network he is located closer to. Note that in this case the networks share the market equally, and net surplus offered to the consumers is the same for both networks. Hence each consumer's decision is optimal, minimizing his transport costs. However, this shared market equilibrium is highly unstable. To see this, consider a slight deviation in consumers' expectations about the market share. This will lead the marginal consumers to switch to the network with the higher market share. As a consequence, expectations are further biased in favor of this network, leading even more consumers to subscribe to it. The outcome of this positive feedback loop between expectations and subscription decisions ultimately leads to all consumers subscribing to one network, resulting in a cornered market. This phenomenon is called market tipping in network economics, and is commonly observed in the presence of positive network...
externalities. We will analyze the dynamics of market shares in more detail in Chapter 5.

Since there are two pure strategies for each consumer, corresponding to subscription to one of the two networks, we obtain the general result (see also LRTb), that generically there are either three consumer equilibria, namely the two cornered market outcomes and an unstable shared market equilibrium, if both $H_1$ and $H_2$ are negative, or a unique, stable consumer equilibrium, which is a cornered market one if $H_1 H_2 < 0$, and a shared market equilibrium if both $H_1$ and $H_2$ are positive. That the number of equilibria is generically three or one is a consequence of the odd-number theorem for equilibria of finite games with generic payoffs. The stability of a unique equilibrium follows from the one-dimensionality of the state space. More on the properties of dynamics in such games can be found e.g. in Hofbauer and Sigmund (1998).

In the following we will assume that the consumer equilibrium is unique, i.e. a stable shared market equilibrium exists. We will see later on, that this assumption is justified if the degree of substitutability between networks is low enough. The attractive features of such an outcome are its uniqueness and its stability. This guarantees, that the extension of the consumers’ game by a pricing stage prior to the subscription decision yields an extensive-form game with a unique subgame perfect equilibrium.

### 2.3.3 Network Equilibria

Imagine prices (including the fixed fee) are set and a corresponding stable consumer equilibrium has been realized. If in this situation neither network can gain by unilaterally changing its prices or fixed fee (taking into account the dependence of consumer equilibria on these values), then these values constitute what we call a network equilibrium. More generally, a network equilibrium is a subgame perfect equilibrium of the game between consumers and both networks, where networks simultaneously set prices in the first stage and consumers simultaneously choose a network to subscribe to in the second stage, having observed the prices set by networks. Since the networks are assumed to be identical in their cost structure, for the remainder of this work we restrict ourselves to symmetric network equilibria. These are network equilibria where the on-net and the off-net prices as well as the fixed fee
of the networks are the same, and the corresponding consumer equilibrium divides the market equally between the two networks, i.e. $\alpha = 1/2$. 
