Chapter 3

Linear Pricing

In this chapter we examine the case of linear pricing, which means that networks do not use a fixed fee, and therefore the total payment for a call is a linear function of minutes called. So in this chapter let $F_1 = F_2 \equiv 0$.

3.1 First Order Conditions

In order to analyze networks’ pricing decisions, we first have to derive their objective function, i.e. the profit functions. For given prices and a corresponding stable consumer equilibrium $\alpha$, profit of network 1 is given by

$$\pi_1 = \alpha^2 (p_{11} - c)q_{11} + \alpha(1 - \alpha)[(p_{12} - c)q_{12} + (a - c_0)(q_{21} - q_{12})],$$

and an analogous equation holds for $\pi_2$.

If we write

$$M_{ij} = [p_{ij} - c(1 + m)]q_{ij} + mcq_{ji}$$

for the unit profit of network $i$ (the profit a single customer of network $i$ generates with one active call to and one passive call from network $j$), denoting by

$$m = (a - c_0)/c > -1$$

the (relative) markup on access, profit of network $i$ can also be written in the form

$$\pi_1 = \alpha^2 M_{11} + \alpha(1 - \alpha)M_{12}.$$
Taking into account that $M_{ii}$ depends only on $p_{ii}$, the first order conditions for a shared market equilibrium are given by

$$\frac{\partial \pi_1}{\partial p_{11}} = 2\alpha \frac{\partial \alpha}{\partial p_{11}} M_{11} + \alpha^2 \frac{\partial M_{11}}{\partial p_{11}} + \frac{\partial \alpha}{\partial p_{11}} (1 - 2\alpha) M_{12} = 0,$$

$$\frac{\partial \pi_1}{\partial p_{12}} = 2\alpha \frac{\partial \alpha}{\partial p_{12}} M_{11} + \alpha (1 - \alpha) \frac{\partial M_{12}}{\partial p_{12}} + \frac{\partial \alpha}{\partial p_{12}} (1 - 2\alpha) M_{12} = 0,$$

and the respective equations for network 2.

At a symmetric shared market equilibrium, where $p_{11} = p_{22}$, $p_{12} = p_{21}$, and $\alpha = 1/2$, the first order conditions for network $i$ read

$$\frac{\partial \alpha_i}{\partial p_{ii}} M_{ii} + \frac{1}{4} \frac{\partial M_{ii}}{\partial p_{ii}} = 0, \quad \frac{\partial \alpha_i}{\partial p_{ij}} M_{ii} + \frac{1}{4} \frac{\partial M_{ij}}{\partial p_{ij}} = 0.$$

Inserting from (2.4), rearranging terms, and with a little abuse of notation treating $\bar{u}(q_{ij})$ as an indirect utility function $\bar{u}(q(p_{ij}))$ of $p_{ij}$, we get

$$\frac{\partial M_{ij}}{\partial p_{ij}} = \frac{\partial M_{ii}}{\partial p_{ii}} \frac{(v' - \bar{u}')(p_{ij})}{(v' + \bar{u})(p_{ii})}, \quad (3.1)$$

$$\frac{\partial M_{ii}}{\partial p_{ii}} = -\sigma M_{ii} \frac{(v' + \bar{u})(p_{ii})}{H_i}. \quad (3.2)$$

What can we infer from these equations about the prices in a stable shared market equilibrium? First, note that $M_{ii}$, the simple unit profit $(p_{ii} - c)q_{ii}$, is positive for $p_{ii} > c$, and upward sloping for $p_{ii} < p^M$, where $p^M$ denotes the monopoly price (for marginal cost $c$)

$$p^M = \operatorname*{argmax}_p \{(p - c)q(p)\}.$$

We also know that $v' + \bar{u}' < 0$ and that $H_i$ must be positive for the shared market equilibrium to be stable. From (3.2) then follows that the unit profit $M_{ii}(p_{ii})$ has the same sign as its derivative. Hence, necessarily, $c < p_{ii} < p^M$. In this sense the equilibrium on-net price is “well-behaved”. This need not be the case for the off-net price. As equation (3.1) suggests, the sign of $\partial M_{ij}/\partial p_{ij}$ depends on the sign of $v' - \bar{u}'$, which may well be positive if marginal passive utility is high.
3.2 Constant Elasticity of Demand

To be a bit more specific, we now invoke the explicit utility function

\[ u(q) = \frac{q^{1-1/\eta}}{1 - 1/\eta}, \]

with \( \eta > 1 \), which was also used by LRTb and yields the constant elasticity demand function

\[ q(p) = p^{-\eta}, \]

indirect utility

\[ u(q(p)) = \frac{\eta}{\eta - 1} p^{1-\eta}, \]

net surplus

\[ v(p) = \frac{1}{\eta - 1} p^{1-\eta}, \]

and a monopoly price of

\[ p^M = \frac{\eta c}{\eta - 1}. \]

Furthermore, for simplicity we assume that the utility from passive calls is a fixed fraction \( \beta \geq 0 \) of the utility from active calls:

\[ \bar{u}(q) = \beta u(q). \]

With these specifications, we can analyze our model in more detail. Inserting in (3.1) and (3.2) and rearranging terms, the first order conditions for network 1 can be expressed by the following system of equations.

\[ p_{12}^{-1} = \frac{1}{1 + m} \left( \frac{1}{p^M} \left( \frac{2\beta \eta}{1 + \beta \eta} + \frac{1 - \beta \eta}{1 + \beta \eta} p_{11}^{-1} \right) \right), \quad (3.3) \]

\[ p_{12}^{-1} = \left[ \frac{p^M}{\eta(p^M - p_{11})p_{11}^{-1}} - \frac{\eta - 1}{2\sigma(1 + \beta \eta)} \right]^{-1}. \quad (3.4) \]

We have intentionally written these equations so as to describe the reciprocal value of the off-net price as a function of the reciprocal value of the on-net
price. This allows us to draw the graphs of the two functions, and find all symmetric candidate equilibria as points of intersection of the corresponding curves.

The next proposition establishes the existence of a unique, stable, symmetric equilibrium for low substitutability. The proof relies on the quasi-concavity of the profit function in the limit as $\sigma \to 0$. It is similar to the proof of Proposition 1 in LRTb. Note, however, that the result is slightly different from LRTb's. Indeed, the call externality prevents the existence of equilibrium in the case of too high substitutability even for $a = c_0$, which is not the case in LRTb's model. For example, if $a = c_0$, i.e. $m = 0$, and $t \to 0$, i.e. $\sigma \to \infty$, the curve given by (3.4) admits its minimum at $p_{11} = p_{12} = c$. For any positive value of $\beta$, however, (3.3) yields $p_{12} > c$ at $p_{11} = c$, and hence there is no point of intersection in the relevant region for large enough values of $\sigma$.

**Proposition 1** For given access charge $a$, if $\sigma$ is small enough, there exists a unique, stable, symmetric network equilibrium. Its price constellation is given by the intersection of (3.3) and the downward sloping part of (3.4).

**Proof:** Consider the case $\sigma \to 0$. Then the networks are monopolies and the prices are at their respective monopoly levels. Graphically, (3.4) degenerates to a vertical line at $p_{11} = p^M$, intersecting (3.3) in $p_{12} = (1 + m)p^M$. This symmetric candidate equilibrium is thus unique and stable (since $H_i = 1/2 > 0$). Moreover, the market shares become constant for $\sigma \to 0$. Hence, given the candidate equilibrium values of $p_{22}$ and $p_{21}$, network 1’s profit is

$$
\pi_1(p_{11}, p_{12}) = \frac{1}{4} [(p_{11} - c)q_{11} + (p_{12} - c(1 + m))q_{12} + mcq_{21}].
$$

This function is quasi-concave in $(p_{11}, p_{12})$, hence $(p^M, (1+m)p^M)$ is its unique maximum. For positive values of $\sigma$ the slope of (3.4) becomes finite, which means that the candidate equilibrium on-net price falls below the monopoly price. The candidate equilibrium remains unique for small values of $\sigma$, and by continuity of $H_i$ in $\sigma$ it remains stable. Also, by continuity of the market share in prices and in $\sigma$, network 1’s profit function remains quasi-concave. Hence the second order conditions are fulfilled for low substitutability. QED
Figure 3.1: Three different positions of the line given by (3.3): (i) $a = c_0$ and $\beta \eta > 1$, (ii) $a < c_0$ and $\beta \eta > 1$, (iii) $a < c_0$ and $\beta \eta < 1$.

In the following we will analyze the graphs of (3.3) and (3.4) more closely, allowing us to derive quickly and easily a variety of comparative statics results.

### 3.3 Graphical Analysis

Let us first have a closer look at (3.3). The right hand side of this equation is an affine linear function of $p_{11}^{-1}$, which depends on the parameters $m$, $\eta$, and $\beta$, but not on $\sigma$. Its slope decreases with $\beta$, falling from $(1 + m)^{-1}$ for $\beta = 0$ to zero for $\beta = 1/\eta$ and approaching $-(1 + m)^{-1}$ for $\beta \to \infty$. At the monopoly price $p_{11} = p^M$, we have

$$p_{12}^{-1} = \frac{1}{(1 + m)p^M},$$

which is independent of $\beta$. Graphically this means that by increasing the relative importance $\beta$ of passive utility, the line in the $p_{11}^{-1}$-$p_{12}^{-1}$-plane given by (3.3) is rotated clockwise around the point $\left(\frac{1}{p^M}, \frac{1}{(1 + m)p^M}\right)$, see Figure 3.1.
Note that without the call externality, i.e. for $\beta = 0$, equation (3.3) reduces to

$$p_{12} = (1 + m)p_{11},$$

the *proportionality rule* from LRTb. For $\beta\eta = 1$, the line (3.3) is horizontal at $p_{12} = (1 + m)p^M$.

If we increase the access charge $a$, and hence the markup $m$, holding all other parameters constant, the line (3.3) rotates clockwise (if its slope is positive, $\beta\eta < 1$) or counterclockwise (if its slope is negative, $\beta\eta > 1$) around the point $\left(\frac{2(\beta\eta - \beta)}{c(\beta\eta - 1)}, 0\right)$, where it intersects the horizontal axis. In both cases the equilibrium moves downwards along the curve (3.4). It cannot reach the horizontal axis, however, since the point of intersection of (3.3) with this axis is always either on the negative side or to the right of $2/c$, i.e. outside the relevant region $c < p_{11} < p^M$. Hence, there is no scope for connectivity breakdown, meaning $p_{12} \to \infty$, contrary to Jeon et al.’s (2004) result for the nonlinear pricing case.

Turning to (3.4), we can see that this equation does not involve $a$, the access charge. Whenever $1/(\eta - 1)$ is not an integer, the right hand side of (3.4) is defined only if the expression in square brackets is nonnegative. The second term of this expression is a negative constant, it does not depend on $p_{11}$. The first term is positive for $p_{11} < p^M$ and -- viewed as a function of $p^{-1}_{11}$ -- downward sloping from its vertical asymptote at $p_{11} = p^M$ to its minimum at $p_{11} = c$, see Figure 3.2. For $p^{-1}_{11} > c^{-1}$ the function given by (3.4) is strictly increasing and unbounded, its slope converging to $\eta^{-1/(\eta - 1)}$ for $p^{-1}_{11} \to \infty$. Furthermore, this function is convex at least for values of $p_{11}$ slightly below $p^M$. The second term in square brackets shifts the curve up (for $\sigma \to \infty$) or down (for $\sigma \to 0$).

Since (3.4) has a negative slope in the relevant region $c < p_{11} < p^M$, there exists at most one point of intersection with (3.3), if the slope of this line is nonnegative, i.e. if $\beta\eta \leq 1$. If $\beta$ exceeds $1/\eta$, the slope of (3.3) is negative, and there exist two points of intersection. However, the second point is outside the relevant region if $\sigma$ is small.

As announced above, we concentrate exclusively on the case where substitutability is low enough to guarantee existence of a unique stable equilibrium.
We then ask, in which way the equilibrium depends on the parameters of the model. All the results are derived using the graphical analysis.

### 3.4 Comparative Statics

The next lemma shows that while the on-net price always decreases with the substitutability parameter $\sigma$, the direction of movement of the off-net price depends on the strength of the call externality and on the elasticity of demand. On the other hand, an increase in the access charge always lowers the on-net price and raises the off-net price.

**Lemma 1**

(i) The on-net price decreases with $\sigma$ and the off-net price decreases with $\sigma$ if $\beta \eta < 1$, increases with $\sigma$ if $\beta \eta > 1$, and is constant at $p_{12} = (1 + m)p^M$ if $\beta \eta = 1$.

(ii) The on-net price decreases in $a$, while the off-net price increases in $a$.

**Proof:** An increase in $\sigma$ shifts the graph of (3.4) upwards and does not influence the graph of (3.3). The point of intersection thus moves to the right, i.e. $p_{11}^{-1}$ increases. The vertical direction of movement depends on the
slope of (3.3). If \( \beta \eta < 1 \) (this includes the LRTb case \( \beta = 0 \)), the slope is positive, so also \( p_{12}^{-1} \) increases. If \( \beta \eta > 1 \) the slope is negative and the intersection point moves down, and if \( \beta \eta = 1 \) the line is horizontal at

\[
p_{12}^{-1} = [(1 + m)p^M]^{-1}.
\]

Increasing \( a \) or, equivalently, \( m \), shifts the line (3.3) downwards. Since (3.4) slopes downward in the relevant region, the point of intersection moves down and to the right. This means \( p_{11} \) falls and \( p_{12} \) rises. \( QED \)

Part (ii) of the lemma appears to contradict a result of LRTb, since the case of no call externality is not excluded. On p. 48 they state that the off-net price may decrease in \( a \) if \( \sigma \) is not small enough, and give a numerical example for this. However, the values they provide \( (\eta = 2 \text{ and } \sigma = c = m = 1) \) lead to the candidate equilibrium prices \( p_{11} = 1 = c \) and \( p_{12} = 2 \). A small increase in \( a \) then does indeed decrease the off-net price, but simultaneously the on-net price falls below marginal cost and in this region any candidate equilibrium is unstable and will therefore not be realized. In the region \( c < p_{11} < p^M \), where the consumer equilibrium is stable, (3.4) is strictly decreasing and hence the off-net price inevitably rises with the access charge.

In contrast to the result in LRTb, more substitutability exerts upward pressure on the off-net price, if \( \beta \) is large enough. Intuitively, if the call externality induced negative effect of an increasing off-net price on the rival's customers is large, higher substitutability creates incentives for the networks to exploit this effect and raise the off-net price while lowering the on-net price to compensate their own customers.

### 3.5 The Collusive Role of the Access Charge

Part (ii) of Lemma 1 states that varying the access charge results in the equilibrium prices moving in opposite directions. We know that the equilibrium on-net price is always below the monopoly price. If this is also the case for the off-net price, the impact on profits of varying the access charge is ambiguous.\(^1\) If, however, the off-net price is above the monopoly price, both

\(^1\)In a symmetric equilibrium, access charges payed and received cancel out. Thus, the relevant monopoly price for off-net calls is based on technical marginal costs \( c \), not on
prices will move towards this monopoly price (and hence raise profits) if and only if the access charge is lowered.

Imagine now that $\beta \eta > 1$. This is not an unrealistic case, since $\eta > 1$ and $\beta$ may well be only slightly below $1$. The slope of (3.3) is then negative, and for $\sigma > 0$ we have $p_{12} > (1 + m)p^M$ in equilibrium. Now let the access charge equal marginal termination cost, so $m = 0$. Then the off-net price exceeds the monopoly price, and we have the situation described above. In order to maximize equilibrium profits, both networks will negotiate an access charge $a$ below $c_0$.

If $\beta \eta = 1$, the equilibrium off-net price is $(1 + m)p^M$, independently of $\sigma$. For $a = c_0$ then $p_{12}$ is at the monopoly level, while $p_{11}$ is below $p^M$. Starting from these values, a small decrease in $a$ raises $p_{11}$ towards the monopoly price and thereby has a positive first-order effect on profits from on-net calls, but only a second-order (negative) effect on profits from off-net calls. In sum, profits rise. By continuity this continues to hold if $\beta \eta$ is not too far below $1$. This shows that networks may prefer an access discount even for $\beta \eta < 1$. For very low values of $\beta$, of course, this need not be the case.

Graphically, this can easily be seen if we keep in mind that since (3.4) is independent from the access charge, networks can only shift the line (3.3) up or down by varying the access charge. Thereby they can select any point on (3.4), subject to the restriction $m > -1$. Maximizing profits, they will choose the point where their isoprofit curve is tangent to (3.4), see Figure 3.3. The point of tangency is unique, at least if $\sigma$ is not too large, since (3.4) is convex in the vicinity of $p_{11} = p^M$ and the equilibrium profit function is quasi-concave in equilibrium prices (the upper-contour sets of the isoprofit curves are convex), peaking at the monopoly point $(1/p^M, 1/p^M)$. It follows immediately that the point of tangency will lie northeast from the monopoly point, as illustrated in Figure 3.4. This means that with the negotiated profit-maximizing access charge, both on- and off-net prices are smaller than the monopoly price. If the slope of (3.3) is negative or only slightly positive, of course, this implies that this line intersects $\{p_{11} = p^M\}$ above the monopoly point. Hence

$$[(1 + m)p^M]^{-1} > (p^M)^{-1},$$

perceived marginal costs $(1 + m)c$, and coincides with the monopoly price $p^M$ for on-net calls.
or $m < 0$. This analysis proves the first part of the next proposition.

**Proposition 2** Fix $\sigma > 0$ small enough. There exists $0 < k < 1$ such that if $\beta \eta > k$, networks will agree on an access discount, if $\beta \eta < k$, networks will negotiate an access markup, and if $\beta \eta = k$, networks will agree on $a = c_0$.

The case $\beta = 0$ is the case without passive utility, and we could in principle just refer to Proposition 2 of LRTb for the proof. In this proposition they state that for small $\sigma > 0$ (and for $\beta = 0$) the profit maximizing access charge exceeds $c_0$. While this statement turns out to be true, unfortunately their proof is flawed. In their proof, LRTb (p. 49) argue that for small $\sigma > 0$ their Lemma 2 shows that both on-net and off-net prices increase with the access charge. From this they infer that starting from $a = c_0$, a small increase in the access charge raises both prices toward the monopoly level and therefore leads to higher profits. However, actually their Lemma 2 (correctly) states that for small $\sigma > 0$ the on-net price decreases in $a$. Hence it is not obvious that an increase in $a$ does indeed raise profits.

In the following we give the correct version of the proof.

*Proof:* It suffices to show that networks will negotiate a markup on access if $\beta = 0$. Given the analysis in the last paragraph, the second and third part
of Proposition 2 then follow immediately from continuity of the negotiated access charge in $\beta \eta$ and from the intermediate value theorem, respectively.

Note, that for $a = c_0$ and $\beta = 0$, the line (3.3) is the diagonal $\{p_{12} = p_{11}\}$. By symmetry of the equilibrium profit function in $p_{11}$ and $p_{12}$, the slope of the isoprofit curves is equal to $-1$ along the diagonal. The slope of (3.4) at the intersection with the diagonal, on the other hand, converges to $-\infty$ as the point of intersection approaches the monopoly point, i.e. as $\sigma \to 0$. Thus, for small $\sigma$ the point of tangency is below the diagonal (see Figure 3.4), where $p_{11} < p_{12}$, and by the proportionality rule, $m > 0$, i.e. a markup on access, is a necessary condition for this. \[QED\]

As noted, for small $\sigma$ the profit maximizing point of tangency lies below the diagonal. Since networks will choose an access charge which lets this point become an equilibrium, we obtain the following corollary:

**Corollary 1** If $\sigma$ is positive but small and networks may cooperatively determine the access charge, then the resulting equilibrium prices will show a markup on off-net calls.

### 3.6 The Socially Optimal Access Charge

From the social viewpoint, the optimal access charge is the access charge that maximizes welfare, the sum of profits and consumer surplus, in equilibrium:

$$W(p_{11}, p_{12}) = \frac{1}{2}[(1 + \beta)u(q_{11}) - cq_{11} + (1 + \beta)u(q_{12}) - cq_{12}]. \quad (3.5)$$

To maximize welfare, the caller would have to be induced to extend the length of his calls up to the point where marginal total utility created equals marginal cost. This means

$$(1 + \beta)u'(q_{ij}) = c$$

and is induced by a price of $p_{ij} = (1 + \beta)^{-1}c$. Of course these prices cannot be sustained in an equilibrium, since they are below marginal cost for $\beta > 0$. Assume a benevolent regulator can set an arbitrary access charge subject to the technical constraint $a > -c_0$. By symmetry, the iso-welfare curves surrounding the unconstrained optimum have a slope of $-1$ along the diagonal.
\{p_{11} = p_{12}\}. Since the slope of (3.4) at the intersection with the diagonal is smaller than $-1$ for small $\sigma$, we can conclude that for small $\sigma$ the point of tangency of (3.4) and the iso-welfare curves lies above the diagonal, and therefore also above the profit maximizing point on (3.4), as shown in Figure 3.4. This means that the welfare maximizing access charge is below marginal cost and also below the profit-maximizing access charge.

Moreover, we can show that the welfare maximizing access charge might actually fall below zero. It follows from the additively separable form of (3.5) that the iso-welfare curves have vertical tangents at $p_{12} = c(1 + \beta)^{-1}$. Since (3.4) becomes vertical at $p_{11} = p^M$ for $\sigma \to 0$, the point of tangency approaches $(1/p^M, (1 + \beta)/c)$. Denoting the socially optimal access charge by $a^w$, this implies that $(1 + a^w - c) p^M$ converges to $\frac{c}{1+\beta}$, or $a^w \to c_0 \left(1 - 2 \frac{1 + \beta \eta}{\eta + \beta \eta}\right)$.

It can be seen that the sign of $a^w$ depends on the relative size of $\beta$ and $\eta$. Note that for $\eta < \frac{2}{1-\beta}$ the expression in brackets is negative, and so is $a^w$. The profit maximizing access charge $a^\pi$, on the other hand, is always positive for small $\sigma > 0$. We summarize this as follows.

Figure 3.4: The socially optimal choice of $a$, illustrated for $\beta \eta > 1$. 
Proposition 3 (i) $a^w < c_0$ for small $\sigma$.
(ii) If $\eta < \frac{2}{1-\beta}$, then $a^w < 0 < a^\pi$ for small $\sigma$.
(iii) If $\eta > \frac{2}{1-\beta}$, then $0 < a^w < a^\pi$ for small $\sigma$.

The more relevant of the cases (ii) and (iii) of this proposition seems to be (ii), especially if we assume that $\beta$ is close to 1. The four different regions in $\beta$-$\eta$-space corresponding to cases $\beta\eta > 1$ and $\beta\eta < 1$ of Proposition 2 and cases (ii) and (iii) of Proposition 3 are shown in Figure 3.5. Note that in this case networks may actually agree on a bill-and-keep arrangement, setting $a = 0$. This might result from the consideration that in existing mobile phone networks, bill-and-keep helps to save transaction costs of interconnection, a point not included in our model. If transaction costs are substantial and were taken into account, bill-and-keep might indeed turn out to be profit maximizing. Note, however, that contrary to the view of Gans and King (2001), from Proposition 3(ii) it follows that bill-and-keep is also welfare improving compared with cost-based access pricing.
3.7 Discussion

In this chapter we introduced call externalities in LRTb’s standard model of network competition with linear prices and termination-based price discrimination. We showed that this has a significant effect on the strategic incentives of network operators.

Corroborating the findings of Gans and King (2001), Dessein (2003), and others, our findings emphasize the point that collusion over the access charge will result in access sold at a discount. Nevertheless, we seem not to encounter this phenomenon in existing mobile phone networks, and regulators are usually struggling with bringing access charges down to cost.

The reason for this might be the nonexistence of a fixed-line network in our models. Indeed, if networks are not allowed to price discriminate in access, high access charges may well be the result of networks’ incentives to boost profits from incoming calls originating on the fixed-line network. Alternatively, as Gans et al. (2005) suggest, even with price discrimination in access, networks may agree to keep mobile-to-mobile access charges at high levels in order to prevent customer arbitrage, i.e. consumers’ substitution of mobile-to-mobile calls with fixed-to-mobile calls. A detailed study of these arguments is beyond the scope of this work.

Recent market research (Horvath and Maldoom, 2002) suggests that there is a strong tendency of mobile telephony to substitute for fixed-line telephony, and some business representatives in the field even believe that voice telephony over fixed-line networks will ultimately disappear completely. If this is true, then the policy implications which can be derived from the present model might indeed turn out to be of strong relevance.