Chapter 4
Nonlinear Pricing

In this chapter we study network competition and compare bill-and-keep with cost-based access pricing within the framework of a simple model where two symmetric networks compete in nonlinear and discriminatory prices in the presence of call externalities. In contrast to Gans and King's result, and corroborating the view of Cambini and Valletti, we argue in favor of bill-and-keep, showing that such an arrangement is indeed welfare improving compared to cost-based access pricing.

Again our analysis is based on the model of LRTb, which was also utilised by Gans and King and Cambini and Valletti. As in Chapter 2, we neglect fixed costs, but extend the model to include call externalities.

Networks now compete in two-part tariffs discriminating between on-net and off-net calls. A customer of network $i$ with volumes $q_{ii}$ of on-net and $q_{ij}$ of off-net calls, respectively, is charged

$$T_i(q_{ii}, q_{ij}) = F_i + p_{ii}q_{ii} + p_{ij}q_{ij},$$

where $F_i$ is a fixed fee and, as usual, $p_{ij}$ is the price for a call from network $i$ to network $j$.

### 4.1 Equilibrium Analysis

Under the assumption of a balanced calling pattern, profit of network $i$ is

$$\pi_i = \alpha_i^2(p_{ii} - c)q_{ii} + \alpha_i\alpha_j[(p_{ij} - c)q_{ij} + (a - c_0)(q_{ji} - q_{ij})] + \alpha_i F_i. \quad (4.1)$$
Remember that the market share \( \alpha_i \) of network \( i \) is determined by the indifferent consumer and given by (2.3). Inserting from (2.1), we get

\[
\alpha_i = \frac{1}{2} + \sigma \alpha_i [v(p_{ii}) - v(p_{ji}) + \bar{u}(q_{ii}) - \bar{u}(q_{ij})] - \\
- \sigma \alpha_j [v(p_{jj}) - v(p_{ij}) + \bar{u}(q_{jj}) - \bar{u}(q_{ji})] + \\
+ \sigma (F_j - F_i).
\] (4.2)

Given the choices of the other network, we can derive the first order conditions in the following way: Imagine that first, for fixed market shares, a network maximizes profits holding its market share constant. It does this by choosing optimal prices \( p_{ii} \) and \( p_{ij} \), while the fixed fee is used to offset deviations of the market share. In a second step, the network chooses its profit maximizing market share. If \( \alpha_i \) is held constant, differentiating (4.2) with respect to \( p_{ii} \) yields

\[
0 = \alpha_i [v'(p_{ii}) + \bar{u}'(q_{ii})q'(p_{ii})] - \frac{\partial F_i}{\partial p_{ii}}.
\]

This can be rewritten as

\[
\frac{\partial F_i}{\partial p_{ii}} = \alpha_i [\bar{u}'(q_{ii})q'(p_{ii}) - q_{ii}].
\] (4.3)

On the other hand, maximizing profit (4.1) with respect to \( p_{ii} \) for constant market shares yields

\[
0 = -\alpha_i [q_{ii} + (p_{ii} - c)q'(p_{ii})] - \frac{\partial F_i}{\partial p_{ii}},
\]

or

\[
\frac{\partial F_i}{\partial p_{ii}} = \alpha_i [(c - p_{ii})q'(p_{ii}) - q_{ii}].
\] (4.4)

Comparing (4.3) and (4.4), we derive the identity \( c - p_{ii} = \bar{u}'(q_{ii}) \), and since \( \bar{u} = \beta u \) and \( u'(q_{ii}) = p_{ii} \),

\[
p_{ii} = \frac{c}{1 + \beta}.
\] (4.5)
Hence the profit maximizing on-net price is always at the social optimum — since \( \bar{u} = \beta u \), this price maximizes \( u(q(p)) + \bar{u}(q(p)) - cq(p) \) — regardless of the market share.

Analogously, we can differentiate with respect to \( p_{ij} \) and compare the expressions for \( \frac{\partial F_i}{\partial p_{ij}} \). This yields

\[
p_{ij} = \frac{(1 - \alpha_i)(c + a - c_0)}{1 - \alpha_i(1 + \beta)}
\]  

(4.6)

for \( \alpha_i < 1/(1 + \beta) \). For \( \alpha_i \to 1/(1 + \beta) \) from below, the optimal off-net price goes to \(+\infty\), i.e. for \( \alpha_i \geq 1/(1 + \beta) \), it is optimal for network \( i \) to deter any off-net call.

In a symmetric shared market equilibrium we have \( \alpha_i = \alpha_j = 1/2 \) and hence

\[
p_{ii}^* = \frac{c}{1 + \beta}, \quad p_{ij}^* = \frac{c + a - c_0}{1 - \beta}.
\]  

(4.7)

As usual when competing in two-part tariffs, networks set prices so as to maximize social welfare, and then extract consumer surplus via the fixed fee. For the on-net price, the call externality is internalized by the network’s pricing decision, while this is not the case for the off-net price. If a network lowers its off-net price, also its rival’s customers benefit through the call externality. In equilibrium this leads to prohibitively high off-net prices if \( \beta \) is large. Indeed, as already noted by Jeon et al. (2004), for \( \beta \to 1 \) the off-net price goes to \(+\infty\), resulting in connectivity breakdown. For the rest of the analysis we assume that \( \beta < 1 \).

### 4.2 Profit Maximizing Access Charge

In a symmetric equilibrium, where \( \alpha_i = 1/2 \) and \( p_{ij} = p_{ji} \), differentiating profit with respect to the fixed fee yields

\[
\frac{\partial \pi_i}{\partial F_i} = \frac{\partial \alpha_i}{\partial F_i} [(p_{ii} - c)q_{ii} + F_i] + \frac{1}{2}.
\]  

(4.8)

From (4.2), \( \partial \alpha_i/\partial F_i \) can be replaced by

\[
\frac{\partial \alpha_i}{\partial F_i} = \frac{\sigma}{2\sigma[v(p_{ii}) - v(p_{ij}) + \bar{u}(q_{ii}) - \bar{u}(q_{ij})] - 1},
\]  

(4.9)
leading to

$$F_i = \frac{1}{2\sigma} - [v(p_{ii}) - v(p_{ij}) + \bar{u}(q_{ii}) - \bar{u}(q_{ij})] - (p_{ii} - c)q_{ii}. \quad (4.10)$$

In a symmetric equilibrium, network profit is given by

$$\pi_i = \frac{1}{4}(p_{ii} - c)q_{ii} + \frac{1}{4}(p_{ij} - c)q_{ij} + \frac{1}{2}F_i. \quad (4.11)$$

Differentiating with respect to $a$ and inserting the equilibrium values of $p_{ij}$ and $F_i$ derived above, yields a profit maximizing access charge which is implicitly given by

$$a^* = c_0 + \frac{(1 - \beta)q(p^*_{ij})}{(1 + 2\beta)q'(p^*_{ij})} - \frac{3\beta c}{1 + 2\beta}. \quad (4.12)$$

Note that since $q'$ is negative, $a^*$ is smaller than $c_0$. Hence networks will invariably negotiate an access charge below marginal cost. Inserting (4.12) into (4.7) yields

$$p^*_{ii} - p^*_{ij} = \frac{1}{1 + 2\beta} \left( \frac{\beta c}{1 + \beta} - \frac{q(p^*_{ij})}{q'(p^*_{ij})} \right) > 0. \quad (4.13)$$

Hence the resulting off-net price is always below the on-net price, independently of $\beta$.

While the off-net price for any given access charge goes to $+\infty$ for $\beta \to 1$, this is not the case for the off-net price resulting from the collusive choice of the access charge — both the nominator and the denominator go to zero at the same rate in the expression

$$p^*_{ij} = (c + a^* - c_0)/(1 - \beta).$$

Intuitively, connectivity breakdown cannot be optimal for networks that are maximizing joint profits.

### 4.3 Welfare Maximizing Access Charge

The socially optimal access charge $a^w$ would be the one giving rise to equilibrium prices

$$p_{ii} = p_{ij} = c/(1 + \beta).$$
From (4.7), this is achieved by

\[ a^w = c_0 - \frac{2\beta c}{1 + \beta}. \]  

(4.14)

Clearly, the socially optimal access charge is below marginal cost. On the other hand, comparing (4.12) and (4.14) yields

\[ a^\pi - a^w = \frac{1 - \beta}{(1 + \beta)(1 + 2\beta)} \left( (1 + \beta) \frac{q(p^*_ij)}{q'(p^*_ij)} - \beta c \right) < 0, \]  

(4.15)

and this means that the profit maximizing access charge is even smaller than the socially optimal one. Summarizing,

\[ -c_0 - c_1 < a^\pi < a^w < c_0. \]

This shows that with two-part tariffs and discriminatory prices, cost-based access pricing can never be optimal from the social viewpoint, if the call externality is taken into account.

If we agree that 'realistic' values of \( \beta \) exceed 1/3, then from (4.14) we always have \( a^w < 0 \). It follows that from the social viewpoint, bill-and-keep, i.e. \( a = 0 \), is a strict improvement over cost-based access pricing. Indeed, bill-and-keep is exactly socially optimal if \( \beta \) happens to equal \( \frac{c_0}{3c_0 + 2c_1} \).

Some authors, e.g. DeGraba (2003), have suggested that the caller and the receiver share the value of a call, i.e. \( \beta \approx 1 \). Since for \( \beta \to 1 \), both \( a^\pi \) and \( a^w \) decrease to \(-c_0 - c_1\), this implies that networks' and regulators' incentives are almost perfectly aligned, eliminating the need for regulatory intervention altogether.

### 4.4 Discussion

In this chapter we studied network competition under a caller-pays system with two-part tariffs and termination-based price discrimination in the presence of call externalities. As in the linear pricing case, it turned out that both the profit maximizing and the welfare maximizing access charges are below marginal cost. Moreover, we made a point for the widespread bill-and-keep arrangements. While we agree with Gans and King that these arrangements
may be a result of tacit collusion, we showed that they are welfare improving compared with cost-based access pricing, corroborating the positive view of Cambini and Valletti. Finally, we demonstrated that if the value of a call is approximately shared between caller and receiver, the need for regulatory intervention tends to vanish altogether.