Chapter 6

The Coexistence Puzzle

6.1 Introduction

In existing mobile telecommunications markets throughout Europe an observable common phenomenon is that on-net prices are considerably below off-net prices. This is due to access charges above marginal cost, the reason for which has been discussed at the end of Chapter 4. At the same time, telecommunications services appear to be very close substitutes. Indeed, voice telephony services offered by different network operators can hardly be argued to show any significant characteristics of horizontal product differentiation. It seems reasonable, therefore, to treat these services as homogeneous. A consequence of this is that the results of the standard models, including their extensions such as in Chapters 3 and 4, are no longer applicable, since a common feature of these models is the nonexistence of equilibrium under close substitutability.

A study of market share dynamics for fixed pricing structures in the absence of product differentiation has been provided in the last chapter. Recall Theorem 1 for the three-networks case, and the analogous analysis for the case of two networks. It is easy to generalize this theorem for an arbitrary number of networks and asymmetric prices (but keeping each network's on-net price below its off-net price).

**Theorem 6** Assume \( n \geq 2 \) symmetric networks compete in the market, behavior of market shares is given by the best response dynamics, and each
network i offers on-net price \( p_i \) and off-net price \( q_i \) with \( p_i < q_i \). Then in the long run a single network covers the market.

**Proof:** Assume the initial state is \( x \in S_n \). The average price \( P_i(x) \) paid by network i’s customers is \( P_i(x) = p_ix_i + q_i(1 - x_i) = q_i - (q_i - p_i)x_i \), which is decreasing in \( x_i \). The network currently offering the lowest average price will grow, and this lowers its average price even further. All other networks lose market share and thereby their average price rises. In the long run then, a single network covers the market. \( QED \)

As we have argued previously, it is a straightforward consequence of on-net prices below off-net prices, that positive tariff-mediated network externalities drive the initially largest network to eventually corner the market. Given that network services are homogeneous, we should expect to see the respective markets to be dominated by a single network. Nevertheless, several networks seem to coexist in these markets for years.

One obvious objection to this “coexistence puzzle” is that real-world markets are not yet in equilibrium. However, anecdotal evidence suggests that not even the movement of market shares is in line with our predictions. In many national markets, large established networks loose market share to newcomers or younger and smaller competing networks. How can this happen? In this chapter we suggest a solution to the coexistence puzzle which relies on the rejection of the balanced calling patterns assumption.

### 6.2 Local Interaction Models

All the models we studied up to now were based on the assumption of balanced calling patterns. Recall that this assumption means that each customer in the market calls each other customer, or at least that he has the same probability of calling each other consumer. This assumption was necessary to allow us to treat the interaction as a population game. For each customer the mixed strategy to best respond to is the average strategy in the population, i.e. the population state. As a consequence, if it is optimal for a customer to choose network \( i \), then this is optimal for all customers, since all are playing the same game. Consider an extreme opposite case, where the whole population is split up into pairs of consumers and each consumer...
exclusively calls his partner. It is clear that with this kind of calling patterns, under the assumptions of Theorem 6, each pair of consumers will coordinate on one of the networks after at least one consumer in each pair has received a switching opportunity. The long run market share of a network depends on the initial state, but can be any fraction in [0, 1].

In real populations, the interaction structure is somewhere between these two extreme cases. While calling patterns will definitely not be balanced in the strict sense, most people typically make phone calls to more than one partner. A quick and non-representative survey among the author's acquaintances suggests that people usually direct a large majority of their private monthly phone call minutes to only a handful of close friends or relatives, while a rather small minority of call minutes terminates in the rest of the population. Such a pattern of interaction is called local, as opposed to the global interaction structure leading to balanced calling patterns. Consequently, evolutionary or social learning models assuming that each agent interacts only with a finite set of other agents are called local interaction models. The game theoretic literature on local interaction models started with Ellisons's (1993) extension of the basic stochastic evolution models of Kandori et al. (1993) and Young (1993). Important results are due to Blume (1993, 1995), Goyal (1996), Berninghaus and Schwalbe (1996), Young (1998), and Ellison (2000). Closely related are recent studies of the contagion effect, see Morris (2000) or Lee and Valentinyi (2000). The economics literature on local interaction is surveyed by Brock and Durlauf (2001). In the following we give a general definition of a local interaction game in the context of telecommunications.

Let \( X \) be a countable population of agents, where each agent \( x \) represents a consumer in a telecommunications market. Let \( \sim \) be a binary relation on \( X \). This relation is meant to describe who makes phone calls to whom. We assume that each agent only calls his "friends", where \( x' \) is a friend of \( x \) if \( x \sim x' \). We assume that friendship is irreflexive, \( x \sim x \), and symmetric, \( x \sim x' \iff x' \sim x \). Thus no agent calls himself, and each agent is the friend of each of his friends. Let \( F(x) \) be the set of \( x \)'s friends. We assume that each agent has at least 1 and at most \( m \) friends, \( 1 \leq |F(x)| \leq m \). Each agent can choose between \( n \) networks in \( N = \{1, \ldots, n\} \), where network \( i \) offers prices \( p_{ij} \) for calls to networks \( j \in N \). Time proceeds in discrete steps \( t = 0, 1, 2, \ldots \), and for simplicity we assume that each agent receives a switching opportunity.
at every time step. The state $S$ of the population is here not the vector of market shares but the exact distribution of networks on the population of agents, i.e. a function $S : X \rightarrow N$, where $N = \{1, \ldots, n\}$, giving for each agent $x$ his network choice $S(x)$. The system starts with an arbitrary initial state $S_0$. At each time $t$ each agent $x \in X$ chooses a best response to $S_t$, i.e. a network $i$ minimizing his total payment $P_t(x, S_t)$:

$$S_{t+1}(x) \in BR(x, S_t) \equiv \arg\min_{i \in N} P_t(x, S_t),$$

where

$$P_t(x, S_t) = \sum_{x' \in F(x)} p_{i, S_t(x')}.$$

A state $S$ is a Nash equilibrium if no agent has an incentive to deviate from his choice. Clearly, $S$ is a Nash equilibrium if and only if $S \in BR(., S)$.

The setting is very general up to now. However, we concentrate on a particularly simple model in order to isolate the differences in long run behavior as compared to the last chapter. Thus we consider only the two-networks case $n = 2$. Moreover, we assume again a symmetric pricing structure $p_{11} = p_{22} = p$ and $p_{12} = p_{21} = q$ with on-net price below off-net price, $p < q$, as in Theorem 6. Finally, we assume that each agent has exactly three friends, i.e. $|F(x)| = 3$ for all $x \in X$. With these specifications, the long run behavior of the population state depends — apart from the initial state — only on the exact graph of the friendship relation on $X$. One particular such graph is exemplified in the next section.

### 6.3 A Simple Hexagonal Graph Structure

Let $X = \mathbb{Z}^2$ and define the following binary relation on $X$:

$$(x_1, x_2) \sim (x'_1, x'_2) \iff \begin{cases} |x'_1 - x_1| = 1 \land x'_2 = x_2 \\ \lor \quad [x_1 + x_2 \text{ is even} \land (x'_1, x'_2) = (x_1, x_2 + 1)] \\ \lor \quad [x_1 + x_2 \text{ is odd} \land (x'_1, x'_2) = (x_1, x_2 - 1)] \end{cases} \quad (6.1)$$

\footnote{An odd number of friends helps to avoid the knife-edge case of indifference between the networks.}
With these specifications each point in the plane with integer coordinates corresponds to an agent, and each agent has three friends: His immediate left neighbor, his immediate right neighbor, and alternatingly his immediate upper or lower neighbor, respectively. The corresponding graph is illustrated in Figure 6.1. Of course this graph is just one particular example. Indeed, even if $X$ is taken to be finite, the size of the set of all graphs meeting the assumptions of the last paragraph grows exponentially in the number of elements of $X$. The graph we study here is particularly simple because of its obvious translation invariance. For each agent $(x_1, x_2)$ with even sum $x_1 + x_2$ we call the set of the six agents

$$\{(x_1, x_2), (x_1 + 1, x_2), (x_1 + 2, x_2), (x_1, x_2 + 1), (x_1 + 1, x_2 + 1), (x_1 + 2, x_2 + 1)\}$$

a hexagon. A hexagon thus consists of the six agents on the boundary of one of the rectangles visible in Figure 6.1.

It is now easy to see the following:

Lemma 2 Let $H$ be a hexagon and assume $S_T(x) = i$ for all $x \in H$. Then $S_t(x) = i$ for all $x \in H$ and all $t \geq T$.

In other words, if all agents in a hexagon use the same network $i$ at time $t = T$, then they will use the same network $i$ at any time $t \geq T$. The reason for this is that each agent $x$ in a hexagon has exactly two friends in the same hexagon. So if all agents in a hexagon use network $i$, then at least $2/3$ of the
friends of $x$ use network $i$. Therefore $x$’s total payment when subscribing to network $i$ is strictly smaller than when subscribing to network $j$, as

$$P_i(x, S_t) = 2p + p_{ik} \leq 2p + q < 2q + p \leq 2q + p_{jk} = P_j(x, S_t),$$

where $k$ is the network used by the third friend of $x$.

This simple result has a deep impact on the nature of Nash equilibria of our local interaction game. The next theorem follows immediately from this lemma and the observation that the complement in $X$ of the union of hexagons is again a union of hexagons.

**Theorem 7** Let $M$ be the union of arbitrary many hexagons in $X$. Define the state $S^*$ by $S^*(x) = i$ for all $x \in M$ and $S^*(x) = j$ for all $x \in X - M$. Then $S^*$ is a Nash equilibrium.

This result says that virtually any distribution of market shares on the two networks can be generated in equilibrium. Of course this plethora of equilibria is devastating for the predictability of market shares, even for given prices.

### 6.4 Discussion

As mentioned before, the graph we studied in this chapter is just one particularly simple example. As such, it does not show some of the characteristics of calling patterns found in real societies, e.g. the “small world” phenomenon (see Watts, 2000). Nevertheless, in its basic structure it comes close to what might be considered the important such characteristics. The decisive assumption here is that a person typically has only a handful of frequent calling partners, whereas only a minority of its call minutes are distributed more or less randomly over the rest of the population. If this person’s friends happen to coordinate on a single network, then the incentive for it to subscribe to the same network is overwhelming, if only on-net prices are significantly below off-net prices. Since it is not unrealistic (albeit ruled out in our model) to assume that small groups of friends will tend to deliberately coordinate

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2It is straightforward to extend this argument to an arbitrary (but odd) number of friends under the symmetry assumptions made: Each agent will join the network the majority of his friends has joined.
in this way, there is considerable room for smaller networks to gain market share, despite the positive feedback loop which would inevitably favor large networks under balanced calling patterns.

The analysis of networks’ strategic price setting behavior under a local interaction structure, however, is made extremely difficult by the multitude of existing consumer equilibria. While we have tried to answer some of the questions arising in the study of the economics of two-way interconnection, many more such questions became apparent. This research issue, therefore, is still far from being settled.
Bibliography


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Band 11 Birgit Trukeschitz: Im Dienst sozialer Dienste. Ökonomische Analyse der Beschäftigung in sozialen Dienstleistungseinrichtungen des Nonprofit Sektors. 2006


www.peterlang.de
Satellite personal communications were expected to form a very successful market, reaching many millions of customers. The market functioned, however it has been, instead, a big business failure. The book examines the rise and fall of the Satellite Personal Communication Systems (SPCS) from an interdisciplinary approach. It describes the technology of a market formed on the basis of heavy investment, which traded international personal communication capacity through the use of mobile satellite networks. Moreover, it analyses the main actors and their business strategies that have lead pioneers to failure. In addition, it focuses on the regulatory efforts, which proved unsuccessful in saving the market from collapsing. The analysis, based on the interaction of various parameters (technology, law, actors and market) forms an interesting level-playing field.

Contents:
- Satellite Personal Communications
- Satellite Technology
- Global Economic Networks
- Strategic Alliances
- Telecommunications
- Legal framework of Satellite Communications
- Business Strategies in Satellite Personal Communications
- Iridium