Essay 1

Robust Multiperiod Poverty Comparisons

Abstract: We propose a methodology for comparing poverty over multiple periods across time and space that does not arbitrarily aggregate income over various years or rely on arbitrarily specified poverty lines or poverty indices. We use multivariate stochastic dominance tests to create dominance surfaces for different time spans. We elaborate the method first for the bidimensional case, using as dimensions income observed over two periods: one at the beginning and one at the end of a time span. Subsequently, we extend it to the case where incomes are observed over $n$-periods. We illustrate our approach by performing poverty comparisons using data for Indonesia and Peru.

based on joint work with Michael Grimm.
1.1 Introduction

Today it is well established that poverty is a dynamic phenomenon. But if poverty does fluctuate and evolve over time, this raises the question of how best to measure it over multiple periods. Cross-sectional poverty measures can provide abundant information on the extent of poverty at a given point, but almost none on the rate at which people escape from or fall into poverty over time.

Recognizing this, authors such as Grootaert and Kanbur (1995) have suggested focusing on households’ changes in poverty status. Others have developed concepts to aggregate incomes over multiple periods (i.e., trajectories of income over time) using an evaluation function that explicitly captures, for example, the risk aversion of households (see e.g., Cruces (2005)). While such an approach has the advantage of accounting for the negative effects of income variability on the well-being of households, it requires arbitrary assumptions about how exactly ‘risk-adjusted mean income’ is best computed.

Likewise, considering the standard spells and component approaches proposed for measuring and conceptualizing chronic and transient poverty, one can safely state that the results and consequently the policy implications depend heavily on how the two forms of poverty are measured: how incomes are aggregated over time, how the poverty line is set, and what poverty index is chosen (see, e.g., Hulme and McKay (2005); Jalan and Ravallion (1998); Duclos et al. (2006a)). Both the spells and component approach usually rely on one specific poverty line and one specific poverty function. Moreover, approaches based on the components approach are usually based on some calculation of average income over time and thus abstract from the exact pattern of the income trajectory. In other words, three consecutive years of high income followed by three consecutive years of low income are treated as six years over which a year of high income follows a year of low income and so on.

To circumvent these problems, we suggest another approach for multiperiod poverty measurement based on stochastic dominance tests. This enables us to establish poverty orderings that are valid for a wide range of aggregation rules of incomes observed over time, a wide range of poverty indices, and a wide range of poverty lines. Our approach relies on the literature on multi-dimensional poverty orderings Duclos et al. (2006b), in which dimensions refer to various indicators of individual well-being such as income, education and health.\(^1\) Our dimensions are incomes observed at different points in time. Defining dimensions in this way raises some further challenges, which we discuss below. We develop our approach first for the case where incomes are observed over two periods and then extend it

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\(^1\)See also Duclos et al. (2006c) and the seminal papers by Bourguignon and Chakravarty (2002, 2003)
to the case where incomes are observed over \( n \)-periods. We illustrate this approach using longitudinal data for Indonesia and Peru. Note that we do not address the issue of income uncertainty and disutility due to income volatility.

Among the papers dealing with multiperiod poverty, probably Hoy and Zheng (2007), Foster (2007) and Bossert et al. (2008) are the closest related to ours.

Hoy and Zheng (2007) suggested a lifetime poverty measure derived from an axiomatic approach. If we computed the poverty measure we suggest for a lifetime period instead for sub-periods of total lifetime, we would be able to derive similar results. However, we do not explore the implications of various axioms one may wish to impose on such a lifetime poverty measure. Moreover, whereas Hoy and Zheng’s approach is designed to compare lifetime poverty across different groups of individuals, our approach is intended to do both, either compare multiperiod poverty across different sub-periods of total lifetime for a given group of individuals or to compare multiperiod poverty for a given sub-period across different groups of individuals.

In the spirit of Hoy and Zheng (2007), Bossert et al. (2008) have also constructed a lifetime poverty measure. Their approach differs in the properties that are deemed relevant. Bossert et al. (2008) model explicitly the persistence of poverty over time by relaxing the notion of path independence considered by Hoy and Zheng (2007). Their index regards the negative effects of being in poverty as cumulative, in the sense that a two-period poverty spell is worse than two one-period spells interrupted by a period out of poverty.

Foster (2007) suggested a new family of chronic poverty measures based on the well-known Foster-Greer-Thorbecke measures (1984). Foster (2007) identifies the chronically poor using two cutoffs: a standard poverty line, which identifies the time periods during which a person is poor, and a duration cutoff, which is the minimum percentage of time a person must be in poverty in order to be chronically poor.

The remainder of our paper is organized as follows. In Section 2 we present our methodology. In Section 3 we implement our methodology empirically and analyze multiperiod poverty in Indonesia and Peru. In Section 4 we discuss our results and conclude.

## 1.2 Methodology

### 1.2.1 Stochastic dominance in a one-period welfare measure

We assume that individual well-being, \( \lambda \), is a function of \( y \), a well-being indicator, for instance income received in period \( t \). Let \( y \) be defined over the interval \([0, \infty]\), where the set of distributions of well-being indicators is \( \Psi := F : [0, \infty] \rightarrow [0, 1] \).
We assume a non-decreasing well-being function without imposing anything concerning the exact contribution of $y$ to well-being:

$$\lambda(y), \text{ where } \frac{\partial \lambda(y)}{\partial(y)} \geq 0.$$ (1.1)

An individual is assumed to be poor if well-being $\lambda(y)$ is below a poverty frontier, $\lambda(z)$, where $z$ is the poverty line belonging to the well-being indicator. The poverty set can then be defined as:

$$\Lambda(\lambda) = \{y|\lambda(y) \leq \lambda(z)\},$$ (1.2)

with $\lambda(z) = 0$.

In what follows we consider, following Atkinson (1987), all additively separable poverty measures $P$ that are non decreasing in $\lambda(y)$ and anonymous. We denote this set of poverty measures $\Xi_1$. Our poverty measure can be computed by:

$$P(F;\lambda) = \int_{\Lambda(\lambda(z))} p(\lambda(y),\lambda(z))dF(y)$$ (1.3)

If well-being is only measured along one dimension, e.g. the one period case, equation 1.3 can be rewritten as:

$$P(F;z) = \int_{0}^{z} p(y,z)dF(y).$$ (1.4)

Our set of poverty measures, $\Xi_1$, includes, for instance, the Watts measure of poverty (Watts, 1968), where $p(y,z) = (\ln z - \ln y)$, and all poverty measures within the Foster-Greer-Thorbecke family, $P_\alpha$ (Foster et al., 1984) with $\alpha \geq 0$ (Foster and Shorrocks, 1988b,c), where $p(y,z) = (1 - y/z)^\alpha$.2

Tests of stochastic dominance are today widely used to establish poverty orderings $D$ that are robust for a broad class of poverty measures, $P(F;z)$, and a large range of poverty lines, $z \in [0, \infty]$.

Given two distributions $F \in \Psi$ and $G \in \Psi$, the first order stochastic dominance condition (FSD), $D_1$, states:

$$FD_1G \forall P \in \Xi_1, z \in [0,z^{\text{max}}] \iff F(z) - G(z) < 0 \forall z \in [0,z^{\text{max}}],$$ (1.5)

where $FD_1G$ means that $F$ has unambiguously less poverty than $G$ with respect to all poverty indices belonging to the class $\Xi_1$ and all poverty lines within the range $[0,z^{\text{max}}]$.

2The Foster-Greer-Thorbecke poverty measure has the formula $P_\alpha = 1/N \sum_{i=1}^{n} (1 - y_i/z)^\alpha$, where $N$ is the total number of individuals $i = 1, \ldots, N$. The parameter $\alpha > 0$ is a poverty aversion parameter: $\alpha = 0$ yields the poverty headcount index, $\alpha = 1$ the poverty gap index, and $\alpha = 2$ poverty severity index (Foster et al., 1984).
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For FSD orderings it is sufficient to compare the distribution function of the well-being indicator in period 1, \( F(y_1) \), with its analog in period 2, \( G(y_2) \). The distribution function can also be called dominance curve. If first order stochastic dominance does not hold, higher-order stochastic dominance tests can be applied to generate robust poverty orderings. Higher-order dominance requires to add further assumptions on how the function \( p(y, z) \) evolves with \( y \). For instance, second order stochastic dominance (SSD) requires to specify \( p(y, z) \) in a way that \( P \) satisfies the Pigou-Dalton transfer principle (see e.g. Foster and Jin (1996)). The Pigou-Dalton transfer principle states that a transfer of income from a richer to a poorer person will not increase poverty as long as that transfer does not reverse the ranking of the two. In this case, the areas under the distribution functions can be compared to generate poverty orderings. If we denote the set of all Daltonian poverty measures, \( \Xi_2 \), then SSD, \( D_2 \), states:

\[
FD_2G \quad \forall P \in \Xi_2, \quad z \in [0, z_{\text{max}}] \iff \int_0^z F(y) \, dy - \int_0^z G(y) \, dy < 0 \quad \forall z \in [0, z_{\text{max}}] \tag{1.6}
\]

where \( FD_2G \) means that \( P(F) \) has unambiguously less poverty than \( P(G) \) with respect to all poverty indices belonging to the class \( \Xi_2 \) and all poverty lines within the range \([0, z_{\text{max}}]\). For instance, within the FGT poverty measure family the poverty gap (\( \alpha = 1 \)) satisfies the Pigou-Dalton transfer principle, the poverty headcount (\( \alpha = 0 \)) does not.

If second order dominance does also not hold, it is possible to integrate the distribution function again and to test for third order dominance. This would of course further limit the set of applicable poverty measures by imposing even more restrictive axioms. Therefore, in the theoretical part of our paper, we restrict our analysis to FSD and SSD. In the empirical part we consider only FSD.

Note also that we do not consider weak stochastic dominance, because statistically it is impossible to distinguish weak and strong stochastic dominance.\(^3\)

It is widely acknowledged that the concept of poverty dominance is useful because it circumvents the problem of choosing one particular poverty measure and one specific poverty line. In the following, we extend the concept, first to two-period welfare measures and then to \( n \)-period welfare measures.

1.2.2 Stochastic dominance in a two-period welfare measure

To take into account the dynamic aspects of poverty, we now extend the one-period well-being function to a two-period well-being function, where the arguments are

\(^3\)Weak stochastic dominance requires \( F(z) - G(z) \leq 0 \) for all poverty measures. Thus strict stochastic dominance, as defined in equation 1.5, implies weak stochastic dominance.
(y_1, y_2), e.g. income received in periods 1 and 2. The well-being function can then be written as:

\[ \lambda(y_1, y_2) : \mathbb{R}^2 \rightarrow \mathbb{R} \mid \frac{\partial \lambda(y_1, y_2)}{\partial y_1} \geq 0, \frac{\partial \lambda(y_1, y_2)}{\partial y_2} \geq 0. \] (1.7)

Hence, we impose the condition that the well-being function, \( \lambda \), is differentiable with respect to the welfare measure in \( t = 1 \) and \( t = 2 \) and that income in both periods contributes positively to individual well-being. Yet, as before, we impose nothing regarding the precise value of the contribution of each year to individual well-being.

We define an individual to be poor if his or her overall well-being \( \lambda(y_1, y_2) \) is below the unknown poverty frontier. In the two-period case, the poverty frontier is not a single point, \( z \), but a locus of points. We define this locus as \( \lambda(y_1, y_2) = 0 \). The overall set of poor people is defined as:

\[ \Lambda(\lambda) = \{ y_1, y_2 | \lambda(y_1, y_2) \leq 0 \}. \] (1.8)

Depending on the specific definition of the locus of the poverty frontier, multiperiod poverty comparisons can be performed according to the ‘intersection’ and the ‘union’ poverty definition (Duclos et al., 2006b). Intersection poverty means in our case that someone is considered poor if well-being is below the poverty threshold in both periods. The concept of ‘intersection’ multiperiod poverty is therefore closely related to the concept of chronic poverty (see e.g. Hulme and Shepherd (2003)). Intersection poverty is represented in figure 1.1 by the crossbred-shaded area under the function \( \lambda_1(y_1, y_2) \) (dashed line). Union poverty means that someone is considered poor if well-being is below the poverty threshold in one of the two periods. This is represented in figure 1.1 by the entire shaded area under the function \( \lambda_2(y_1, y_2) \) (dotted line). In the empirical part of our paper we emphasize the parallels with the concept of chronic poverty and thus focus on intersection poverty.

As in the one-period case, we consider all additively separable, non-decreasing and anonymous poverty measures \( P \). However, we add a further restriction. We require \( y_1 \) and \( y_2 \) to be substitutes in \( \lambda(y_1, y_2) \).\(^4\) This assumption implies that an increase of the well-being indicator in one period increases well-being more the lower the well-being indicator in the other period. Hence, our concept of multiperiod poverty accounts for the correlation between individuals’ outcomes across both periods. We denote this set of poverty measures \( \Xi_{1,1} \). Transferring equation 1.4 to the two-period case, the poverty measure reads:

\[ P(F; \lambda) = \int \int_{\Lambda(\lambda(z_1, z_2))} p(\lambda(y_1, y_2), \lambda(z_1, z_2)) dF(y_1, y_2). \] (1.9)

\(^4\)Specifically, we impose:

\[ \frac{\partial^2 \lambda(y_1, y_2)}{\partial y_1 \partial y_2} \geq 0, \forall y_1, y_2. \]
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Figure 1.1: Test Domain for Dynamic Poverty Comparisons

Equation 1.9 holds for multiperiod poverty comparisons according to the intersection as well as according to the union definition of poverty, depending on the locus of $\Lambda(\lambda)$. If we focus on intersection poverty, as we will do in the empirical part, equation 1.9 could be rewritten as:

$$P(F;z_1, z_2) = \int_0^{z_1} \int_0^{z_2} p(y_1, y_2, z_1, z_2) dF(y_1, y_2).$$  (1.10)

Obviously, as for usual period-by-period poverty orderings, it is desirable that poverty orderings over multiple time spans, $T_j$, are robust to a large set of poverty lines $z \in Z$. This can be ensured by simply transferring the concept of stochastic dominance for univariate welfare distributions to the case of bivariate welfare distributions. A comparison of two time spans is denoted in what follows as $T_a = [t_{1a}; t_{2a}]$ vs. $T_b = [t_{1b}; t_{2b}]$, where $t$ now has an index for the period within each time span, year 1 or year 2, and an index for the time span, time span $a$ or time span $b$.

Furthermore, poverty orderings in the bivariate case, i.e. across time spans, should be robust to a broad range of procedures to aggregate the observed period-specific well-being indicators over the two periods constituting a time span. Thus, the weight given to each single period should not matter, i.e. whether we weigh each period equally or whether we give to one period a higher weight than to the
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other. A reason for doing the latter might be to account for time preference, i.e. one weighs income today more than income tomorrow. Hence, we require that our ordering is robust to the magnitude and even the sign of the time discount rate.\(^5\)

Given two distributions \(F(y_{1a}, y_{2a}) \in \Psi\) and \(G(y_{1b}, y_{2b}) \in \Psi\) the first order stochastic dominance condition, \(D_{1,1}\), states:

\[
FD_{1,1}G \forall P \in \Xi_{1,1}, z_1 \in [0, z_1^{\text{max}}], z_2 \in [0, z_2^{\text{max}}] \\
\iff F(z) - G(z) < 0 \forall z_1 \in [0, z_1^{\text{max}}], z_2 \in [0, z_2^{\text{max}}], \tag{1.11}
\]

where \(FD_{1,1}G\) means that multiperiod poverty is lower over time span \(T_a\) than over time span \(T_b\) with respect to all poverty indices belonging to the class \(\Xi_{1,1}\) and all poverty within the range \([0, z_1^{\text{max}}]\) and \([0, z_2^{\text{max}}]\).

As in the one-period case, tests of higher order dominance could be equally well established by imposing further assumptions regarding the effect of \(y\) on \(p(y_1, y_2, z_1, z_2)\). For instance, holding constant the distribution in period 2, we could impose that a transfer from a richer to a poorer person in period 1 reduces poverty. Symmetrically, we would then impose the same transfer sensitivity on period 2.

As mentioned above, we also require our concept to be robust to a broad range of procedures for aggregating the observed period-specific well-being indicators over the two periods constituting a time span. The simplest way to deviate from an aggregation where each period receives the same weight is to vary the poverty lines within time spans, since this varies the income necessary to be beyond the period-specific poverty frontier in each period. If we chose \(z_1 \neq z_2\) s.t. \(z_{1a} = z_{1b}\) and \(z_{2a} = z_{2b}\), i.e., to give a different weight to the first and second period each time span, the test domain for intersection poverty dominance represents a rectangle, where \(y_1 < z_1\) and \(y_2 < z_2\). This is illustrated the dashed line in Figure 1.1. In what follows, the aggregation procedure is incorporated through the definition of the poverty lines.

In our methodology, and in contrast to ‘one-period-stochastic-domiance’, \(F(y_1, y_2)\) refers now to a bivariate distribution. Hence, the test of stochastic dominance does not imply comparing two curves, as with one-period well-being measures, but two surfaces, where each surface is characterized by its two periods – the well-being measure in the first and second period – and the cumulative density at each point of that surface. Rewriting equation 1.10 shows that the dominance

\(^5\)In a similar way one could account for uncertainty regarding the right way to deflate incomes from one period to the next.
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surface is the product of the two unidimensional curves plus the covariance in the poverty indices in the two dimensions (Duclos et al., 2006b):

$$P(F; z_1, z_2) = \int_0^{z_1} p(y_1, z_1) dF(y_1) \int_0^{z_2} p(y_2, z_2) dF(y_2) + \text{cov}[(p(y_1, z_1)), (p(y_2, z_2))]\quad (1.12)$$

The higher the correlation of individuals’ incomes the ‘higher’ the dominance surface. Our multiperiod poverty index therefore implicitly judges a situation in which one individual is always poor and one always rich worse, ceteris paribus, than a situation where two individuals are poor in one period and rich in the other period. A further comment regarding the robustness to the aggregation procedure is in order. In fact, their way we deal with this problem implies that there is one special situation in which robustness to the aggregation procedure cannot be tested. This arises when the time spans under consideration overlap, i.e., when the second period of the first time span simultaneously represents the first period of the second time span. For instance, if poverty over the time span 1980-1990 has to be compared with poverty over the time span 1990-2000, i.e. $y_2 = y_1$. In this case, the same weight has to be assigned to each.

In this special case the dominance criteria simplifies to:

$$FD_{1,1} G \forall P \in \Xi_{1,1}, z \in [0, z_{\text{max}}] \iff F(z, z) - G(z, z) < 0 \forall z \in [0, z_{\text{max}}] \quad (1.13)$$

where $FD_{1,1} G$ means that multiperiod poverty is lower time span $T_a$ than over time span $T_b$ with respect to all poverty indices belonging to the class $\Xi_{1,1}$ and all poverty lines within the range $[0, z_{\text{max}}]$. Note that we now only test dominance between the two surfaces along an expansion path of $z$, where $y_1 < z$ and $y_2 < z$ (see the bisector line in figure 1.1).

The ‘overlap’ problem can obviously not occur with comparisons over space, say, if poverty over the timespan 1990-2000 in country a is compared to poverty over the timespan 1990-2000 in country b. In this case nothing prevents us to choose $z_1 \neq z_2$ s.t. $z_1 = z_1$ and $z_2 = z_2$, i.e. to give a different weight to the first and second period within each time span in each country.

1.2.3 Stochastic dominance in a $n$-period welfare measure

Extending our methodology to the $n$-period case is straightforward. Our well-being measure becomes $\lambda(y_1, y_2, \ldots, y_n)$. The well-being measure is differentiable with respect to each single period income $y_i$, where $\partial \lambda(y_1, y_2, \ldots, y_n)/\partial y_i \geq 0$. The poverty locus becomes a $n$-dimensional space.

Given two distributions $F(y_{1a}, y_{2a}, \ldots, y_{na}) \in \Psi$ and $G(y_{1b}, y_{2b}, \ldots, y_{nb}) \in \Psi$ the first order stochastic dominance condition, $D_{1,1,\ldots,1}$, states:

$$FD_{1,1,\ldots,1} G \forall P \in \Xi_{1,1,\ldots,1}, z_1 \in [0, z_{1\text{max}}], z_2 \in [0, z_{2\text{max}}], \ldots, z_n \in [0, z_{n\text{max}}]$$
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\[ F(z_1, z_2, \ldots, z_n) - G(z_1, z_2, \ldots, z_n) < 0 \]

\[ \forall z_1 \in [0, z_{1, \text{max}}], z_2 \in [0, z_{2, \text{max}}], \ldots, z_n \in [0, z_{n, \text{max}}] \] (1.14)

where \( F_{D1,1,\ldots,1}G \) means that multiperiod poverty is lower over time span \( T_a \) than over time span \( T_b \) with respect to all poverty indices belonging to the class \( \Xi_{1,1,\ldots,1} \) and all poverty lines within the range \( z_1 \in [0, z_{1, \text{max}}], z_2 \in [0, z_{2, \text{max}}], \ldots, z_n \in [0, z_{n, \text{max}}] \).

Of course the \( n \)-dimensional case allows us again to be robust with respect to the aggregation procedure by giving a different weight to the \( n \) periods within each time span, i.e. by choosing \( z_1 \neq z_2, \ldots, z_{n-1} \neq z_n \) s.t. \( z_{1a} = z_{1b}, z_{2a} = z_{2b}, \ldots, z_{na} = z_{nb} \). \( F(y_1, y_2, \ldots, y_n) \) now refers to a \( n \)-variate distribution and, hence, the test of stochastic dominance now implies comparing two hypersurfaces, where each hypersurface is characterized by its \( n \) dimensions – the welfare measure observed over the \( n \) periods – and the cumulative density at each point of that hypersurface.

An additional issue that arises in the \( n \) period case is how exactly the two time spans are compared. Theoretically, one can compare time spans built using different sets of periods as long as each time span has the same number of periods and as long as the beginning and the end of the first time span each precede the beginning and the end of the second time span respectively. One can then even test for dominance over all these comparisons. Below we illustrate such a case using time spans of a maximum length of four years.

1.2.4 Relative poverty comparison

So far we have proposed the methodology of multiperiod poverty comparison for the concept of absolute poverty. Absolute poverty measures deal with income mobility; they consider an absolute poverty frontier and keep track of people who either stay below or cross this fixed frontier. However, the methodology of multiperiod poverty comparisons is equally well applicable to the concept of relative poverty. Relative poverty measures take into account social mobility; while still keeping track of people who either stay below or cross the poverty line, this frontier becomes endogenous, for example, expressed as a ratio of the median income. Embedding our concept of multiperiod poverty in the concept of relative poverty has some common features with the concept of ‘social exclusion’ as formulated by Bossert et al. (2007).

1.2.5 Estimation and inference

To establish first order stochastic dominance empirically, it is sufficient – as shown by Duclos et al. (2006b) – to calculate the differences of \( \hat{F}(y_{1a}, y_{2a}, \ldots, y_{na}) \) and \( \hat{G}(y_{1b}, y_{2b}, \ldots, y_{nb}) \) on a sufficiently narrow grid of test points and to test the statistical significance of these differences based on student \( t \)-tests (where *refers to
estimated values). The relevant test domain changes based on the definition – union or intersection – of poverty.

1.2.6 Bounds to multidimensional dominance

When applying the methodology presented above, one needs to define a maximum poverty set \( \lambda^* (z_1, z_2, \ldots, z_n \in Z) \). Obviously, defining that frontier is always arbitrary. We again follow Duclos et al. (2006b) and estimate that frontier directly from our sample as the maximum \( \lambda^+ \) for which multiperiod poverty dominance holds. Then we can locate within \( \lambda^+ \) all possible poverty frontiers for which there is necessarily more poverty in time span \( a \) than in time span \( b \). We then can judge on a case-by-case basis whether these critical sets and frontiers are wide enough to justify the conclusion on poverty dominance.

1.3 Empirical illustration

1.3.1 Data

To illustrate the methodology presented above, we use longitudinal data for Indonesia and Peru.

For Indonesia, we use all three existing waves of the Indonesian Family Life Survey conducted by RAND, the University of California Los Angeles, the University of Indonesia’s Demographic Institute and the Center for Population and Policy Studies of the University of Gadjah Mada in 1993 (IFLS1), 1997 (IFLS2) and 2000 (IFLS3). The IFLS is representative of 83% of the Indonesian population living in 13 of the (at that time) nation’s 26 provinces. The IFLS is judged as having a very high quality, among other things, because individuals who have moved are tracked to their new location and, where possible, interviewed there (for details see Strauss et al. (2004)). Using the three waves, we built two panels one from 1993 to 1997 and one from 1997 to 2000, each comprising roughly 32,000 individuals living in 7,000 households. We use real household expenditure per capita as the welfare measure, but refer to it as income in the following. Expenditure is expressed in 1993 prices and adjusted by regional price deflators to the Jakarta price level.

For Peru we use six waves (1997-2002) of the yearly Peruvian Encuesta Nacional de Hogares conducted by the Instituto Nacional de Estadística e Informática. The ENAHO is representative for the three rural and four urban areas of Peru. The ‘panel-households’ are only a sub-sample of all households interviewed. Each year, some households drop out of the panel and others are added (rotating panel). We construct several year-to-year panels, each containing, with...
a few exceptions, more than 5,000 individuals living in more than 1,000 households. We use again real household expenditure per capita as the income measure. Expenditure is expressed in 2002 prices and adjusted by regional price deflators to the Lima price level.

To make income comparable between Indonesia and Peru we convert local currencies to international USD. Purchasing Power Parities (PPP) were taken from the Penn World Table 6.1 (see Heston et al. (2002)).

1.3.2 Robust multiperiod poverty comparisons for the two-period case

In the following we first show empirically how to test for robustness to poverty lines. In this case the arbitrary poverty line is assumed to be constant across the $n$ periods. We then show how to test for robustness to the aggregation procedure by using different poverty lines across periods. To keep the exposition simple and short the empirical illustration will primarily focus on first order stochastic dominance tests using the intersection definition of poverty.

Robustness to poverty lines

To analyze the robustness to the poverty line we use three waves of the Peruvian household panel data and consider the time spans 1998 to 1999 and 1999 to 2000. According to equation 1.13, for order stochastic poverty comparisons can be made by testing for significant differences between the dominance surface of 1998/99 and the dominance surface of 1999/2000. Testing robustness to the poverty line implies testing all points on the bisector between income in period 1 and income in period 2. Figure 1.2 shows the dominance surface of the first time span 1998-1999. The $x$ and $y$ axes measure income (or more precisely household expenditure per capita per day) at the beginning (1998) and the end (1999) of the time span. Expenditures are expressed in 2002 US$ PPP equivalents. The third axis measures the cumulative share of individuals who are below the points defined in the $(x,y)$ domain.

Figure 1.3 shows the difference between the dominance surfaces of the time spans 1999/98 and 1999/2000. The relevant points can be found on the bisector of the graph, since we are testing only robustness to the poverty line (i.e., $z_1 = z_2$). The figure shows that for very low incomes, multiperiod poverty was higher in the first than in the second time span for all poverty indices belonging to the class $\Xi_{1,1}$. However, as we increase the poverty line, we find that the cumulative share of people having had an income below that poverty line increases faster and that multiperiod poverty becomes higher for the second time span. This is a very
Figure 1.2: Poverty in Peru: Dominance Surface of the Time Span 1998/99

Income: Household income per capita per day in PPP US$

Source: Authors’ calculations based on ENAHO

interesting result because it highlights the importance of conducting dominance tests in this context. It can be seen even more clearly in Table 1.1.

The vertical axis in Table 1.1 shows income at the beginning of the time spans and the horizontal axis at the end of the time spans. The value ‘1’ indicates a significant positive difference, i.e., 1999/2000 dominates 1998/99. ‘0’ means an insignificant difference, while ‘−1’ indicates a significant negative difference, i.e., 1998/99 dominates 1999/2000. Actually, we should check for poverty dominance at every possible point on this bisector, i.e. at every possible poverty line (e.g., $1, $1.01, $1.02, etc.). However, to keep the presentation simple and transparent, we abstained from such a detailed analysis and report results only at all poverty lines that are multiples of $0.5. Again, the table demonstrates the relevance of our approach. Relying on the $1 poverty line, one can conclude that ‘chronic’ poverty, i.e. individuals who are under the poverty line in both periods constituting a time span, would have fallen from the first to the second time span because there were more individuals with less than $1 in 1998 and 1999 than in 1999 and 2000. However, if we rely on the $2 poverty line, dominance does not hold anymore given the insignificant differences between the surfaces. Finally, if we rely on the $3 poverty line, one can conclude that chronic poverty has risen from the first to the second time span. Thus, any conclusion about poverty orderings relies heavily on the poverty line chosen. In other words, to state that ‘chronic’ poverty
Figure 1.3: Poverty in Peru: Difference in Dominance Surfaces (1998/99 - 1999/2000)

Income: Household income per capita per day in PPP US$

Source: Authors’ calculations based on ENAHO


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Income: Household income per capita per day in PPP US$. 1 indicates that the 1998/99 surface was significantly above the 1999/2000 surface, −1 indicates the opposite, 0 indicates no significant difference. Significance level: 5%

Source: Authors’ calculations based on ENAHO
as defined here) has changed significantly from one time span to another, one has first to define an appropriate maximum poverty line and then check whether poverty dominance holds at every possible poverty line up to this maximum.

Robustness to Aggregation Procedures

Robustness to the aggregation procedure seems to be equally important since the weights attributed to different periods are often arbitrary chosen. ‘Time discounting’, for instance, might appear to be the most appropriate weighting scheme for economists. However, it is empirically very difficult to obtain a reliable and precise value for consumers’ discount rates. One therefore needs to be sure that the poverty ordering is robust against alternative weights in a reasonable range. Variations in the discount rate mean changes in the aggregation procedure across periods within a time span. Again, as mentioned above, we chose here a very simple way in attributing different weights to different periods. We simply apply different poverty lines to period 1 and period 2 within each time span. In other words, applying a higher poverty line in the second period than in the first period has the same effect than applying a discount rate to period 2 poverty. As will be demonstrated now (if the time spans under consideration do not overlap) our methodology simultaneously ensures robustness to poverty lines and aggregation procedures. Moreover, it ensures of course also robustness to a wide range of poverty measures.

We compare the time span 1998/1999 with the time span 2000/2001. In contrast to the procedure illustrated above, now one has not only to check for significant differences between the two surfaces at the bisector but at all points below and above the bisector up to a reasonable maximum poverty line. This becomes clear when looking at Figure 1.4 and Table 1.2. Figure 1.4 shows the difference between the two dominance surfaces. A robust poverty ordering would require that one surface is above the other surface at all points up to a reasonable maximum poverty line. This is obviously not the case here. Table 1.2 illustrates this further. Given the many ‘0’s’ in the grid of test points, it is clear that poverty dominance cannot be established for any reasonable set of poverty lines in any aggregation procedure.

To underline the economic relevance of our approach, we now show the specific outcomes of weighting period 1 and period 2 differently. We consider poverty orderings $D$ which are robust for a broad class of poverty measures, $P(F; z; r)$ and a large range of poverty lines, $z \in Z$ and discount rates, $r \in R$. Hence, we rely on a poverty index $P$ that assesses the degree of poverty, given a two-period distri-
Figure 1.4: Poverty in Peru: Difference in Dominance Surfaces (1998/99 - 2000/01)

Income: Household income per capita per day in PPP US$

Source: Authors’ calculations based on ENAHO

Table 1.2: Poverty in Peru – Difference in Dominance Surfaces (1998/99 - 2000/01)

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Income: Household income per capita per day in PPP US$
1 indicates that the 1998/99 surface was significantly above the 2000/01 surface, −1 indicates the opposite, 0 indicates no significant difference. Significance level: 5%

Source: Authors’ calculations based on ENAHO
bution $F(y_1,y_2)$ when the poverty line is $z$ and the discount factor of subsequent periods to the first period of a given time span is $r$. Therefore, we state that:

$$F \mathbf{D}_1 G \forall P \in \Xi_{1,1}, z \in [0,z_{\text{max}}], r \in [0,r_{\text{max}}]$$

$$\iff F(z,z(1+r)) - G(z,z(1+r)) < 0 \forall z \in [0,z_{\text{max}}], r \in [0,r_{\text{max}}]$$  \hspace{1cm} (1.15)$$

where $F \mathbf{D}_1 G$ means that multiperiod poverty is lower over time span $T_a$ than over time span $T_b$ with respect to all poverty indices belonging to the class $\Xi_{1,1}$, all (time-constant) poverty lines within the range $[0,z_{\text{max}}]$ and any weighting factor in the range $R$ to discount incomes observed in later periods to the first period constituting a time span.

To illustrate this methodology, we consider the comparison of the time spans 1998/99 and 2000/2002. The two time spans are of different length, such that discounting to the present may be important.\(^6\) The results are shown in Figure 1.5 and Table 1.3. Table 1.3 has two dimensions. The first dimension corresponds to income, and the second corresponds to the discount rate used. That means that each cell corresponds to one point of the bisector between income in the first and second period of each time span, where income in the second period is discounted by the factor $(1+r)^{-n}$, where $n$ is the length of the respective time span measured in years. For instance, the ‘1’ in the sixth column of the first row means that if incomes of period 2 in each time span are discounted by a factor 1.05 per year, multiperiod poverty was significantly higher in 1998/99 than in 2000/01. As before, we check at a grid of test points for significant differences of the bisectors for a large range of discount rates and poverty lines. Overall, Table 1.3 shows that in this comparison, 1998/99 vs. 2000/2002, poverty dominance does hold up to a poverty line of $\$1.2$ and a discount rate of $r = 0.05$, but not beyond.

**Comparisons across socio-economic groups**

Another meaningful example for our proposed concept is to compare multiperiod poverty across groups, e.g. socioeconomic categories, within a country. Comparing poverty of employees in the formal private sector with poverty of self-employed individuals in the informal sector based on multiperiod stochastic dominance could yield different findings than a simple comparison on cross-section comparisons or multi-period average based comparisons. As before, differing results may occur depending on the chosen poverty line and the time-discount rate.

This is now illustrated for Indonesia and the time span 1993/1997. We ask whether intersection poverty was more severe for self-employed than for private

\(^6\)One might also argue that past poverty may be more important than present poverty. Hence, it could also be useful to consider negative instead of positive discount rates.
Figure 1.5: Poverty in Peru: Difference in Dominance Surfaces (1998/99 - 2000/02)

Income: Household income per capita per day in PPP US$

Source: Authors’ calculations based on ENAHO

Table 1.3: Poverty in Peru – Difference in Dominance Surfaces (1998/99 - 2000/02)

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Income: Household income per capita per day in PPP US$; 1 indicates that the 1998/99 surface was significantly above 2000/02 surface, −1 indicates the opposite, 0 indicates no significant difference. Significance level: 5%; Source: Authors’ calculations based on ENAHO
1.3. EMPIRICAL ILLUSTRATION

Table 1.4: Poverty in Indonesia – Differences in dominance surfaces (Self-employed - Private sector)

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Income: Household income per capita per day in PPP US$; 1 indicates that the 1993/97 surface of the self-employed was significantly above the 1993/1997 surface of the private sector employees, –1 indicates the opposite, 0 indicates no significant difference. Significance level: 5%

Source: Authors’ calculations based on ENAHO

sector employees, or vice versa, regardless of the chosen poverty line and aggregation procedure. The results are displayed in Table 1.4. Since there are only few private sector employees with an income below $1 per person and day, the grid starts at the $1.25 poverty line. Ignoring the issue of the aggregation procedure, the findings demonstrate poverty dominance of private sector employees over self-employed up to a maximum poverty line of $3.25. No matter what poverty line up to a poverty line of $3.25 is chosen, one finds more self-employed individuals below the poverty line. If discounting is introduced, this result can be approved almost over the entire grid, except for choosing the $1.25 poverty line. If the $1.25 poverty line is chosen, discounting income in 1997 to the present value of 1993 could render the differences between the dominance surfaces insignificant – as shown by the two ‘0’s’ in line 1. However for any other poverty line between $1.5 and $3 poverty dominance holds regardless of the applied discount rate.

1.3.3 Robust multiperiod relative poverty comparisons for the two-period case within and across countries

We now apply our concept to relative poverty comparisons. To illustrate the idea of relative poverty, consider a household that has experienced a significant increase in income from one period to another and thus moved out of poverty from...
an absolute perspective. If the income of almost all households in the region has risen in a similar way, this household might still be poor from a relative perspective, i.e., the poverty gap to the median did not decline. Accordingly, people are referred to as ‘chronically poor’ in relative terms if their income, measured as a ratio of the median income, stays below a given proportion for consecutive years.

To test for differences in relative poverty between two time spans, we standardize household expenditures by a relative poverty line $\tilde{z}$, i.e., $\tilde{y} = y / \tilde{z}$. We choose $\tilde{z} = 50\%$ of median income. Accordingly, a relative income of 1, for example, means that the individual’s income is exactly half of the median income.

To illustrate the concept of relative multiperiod poverty, we compare two time spans in Indonesia, namely the time spans 1993/97 and 1997/2000. The difference in relative poverty between these two time spans is presented in Figure 1.6 (note that incomes are standardized to 50% of the median income, i.e. a value of 0.8 corresponds to 40% of the median). The $x$ and $y$ axes measure relative income, $\tilde{y}$, at the beginning and the end of the time spans. The figure does not show any systematic pattern. This is supported by Table 1.5, which shows the grid of test points. Here the 0 in the third row of the third column, for example, means that the share of individuals who had less than 50% of the median income ($\tilde{y} = 1$) did not significantly change between the time spans 1993/97 and 1997/2000. Hence, no conclusions about changes in multiperiod poverty can be drawn.

Our concept of relative poverty orderings is also applicable to cross-country comparisons. Absolute poverty comparisons using some agreed international poverty line are interesting if countries have comparable and rather low living standards. But for countries with very different living standards or for very rich countries, relative poverty might be more relevant. To illustrate this, we now compare Peru to Indonesia. Peru has a median income of 4.7$ PPP and Indonesia of 3.7$ PPP per person per day. For these two countries, we consider the time span 1997/2000 with income observations in 1997 and 2000 for each.

Table 1.6 shows the matrix of test points of differences of the two-period poverty surfaces (‘Peru minus Indonesia’). Relative poverty is higher in Peru. Even though dominance cannot be established over the entire domain, the maximum poverty set for relative dynamic poverty is wide enough to conclude dominance. The proportion of poor individuals is higher in Peru no matter what ‘reasonable’ relative poverty line or aggregation procedure is chosen.
Figure 1.6: Relative poverty in Indonesia: Difference in Dominance Surfaces (1993/97 - 1997/2000)

Income: Household income per capita per day in PPP US$

\[ \text{Income} \] is household income per capita per say in US$, standardized by a relative poverty line, \( \tilde{z} = 50\% \) of median income: \( \tilde{\text{Income}} = \text{Income} / \tilde{z} \); 1 indicates that the 1993/97 surface was significantly above the 1997/2000 surface, \(-1\) indicates the opposite, 0 indicates no significant difference. Significance level: 5%; \textit{Source: Authors’ calculations based on IFLS}
1. ROBUST MULTIPERIOD POVERTY COMPARISONS


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*Income* is household income per capita per say in US$, standardized by a relative poverty line, \( \tilde{z} = 50\% \) of median income: *Income* = *Income* / \( \tilde{z} \); 1 indicates that the Peru surface was significantly above the Indonesia surface, -1 indicates the opposite, 0 indicates no significant difference. Significance level: 5%; *Source: Authors’ calculations based on ENAHO and IFLS*

### 1.3.4 Robust multiperiod poverty comparisons for the *n*-period case

Obviously, poverty comparisons over two time spans demand panel data over multiple periods. Consequently, the question arises how the time spans under consideration should be constructed if more than two periods are available within each time span. Which period should be the end of the first and the beginning of the second time span? How many periods should constitute a time span? These are very general questions regarding the measurement of multiperiod poverty (or chronic poverty more specifically). Depending on the panel data available, often several different time span constructions are possible, varying in time span length and the number of periods taken into account. This raises the question, for example, whether comparisons should be made with the maximum overlap (e.g., \( T_a[y_1, y_2, \ldots, y_{n-1}] \) vs. \( T_b[y_2, y_3, \ldots, y_n] \)), without any overlap (e.g., \( T_a[y_1, y_2, \ldots, y_{n/2}] \) vs. \( T_b[y_{n/2+1}, y_{n/2+2}, \ldots, y_n] \)), or with something in between. Depending on these choices, poverty orderings may differ. Thus, beyond robustness to poverty indices, poverty lines and aggregation procedures, one may also require poverty comparisons to be robust to the construction of the time spans.
To illustrate this, we use five waves of the Peruvian household panel data (1998-2002). To simplify the exposition, we require that in each comparison, the first period of time span $T_A$ is 1998 and the last period of time span $T_B$ is 2002. We also abstain from making comparisons for different time span lengths. However, all remaining decisions regarding the construction of these time spans are arbitrary and consequently, any poverty ordering may depend on how exactly the construction is carried out. We think, there are at least five different comparisons that make sense from an economic point of view: three where we consider time spans comprising two periods, one where we consider time spans comprising three periods, and one where we consider time spans comprising four periods:

\[
\begin{align*}
\end{align*}
\]

Given the difficulty in determining which of these five comparisons is most appropriate, dynamic poverty comparisons should be robust to all of them. For example, one can imagine a case in which the $3 poverty line is considered to be a reasonable maximum poverty line when comparing poverty dynamics for the time span 1998-2002 in Peru. In this case, the poverty ordering is only considered robust if poverty dominance can be established for every possible poverty line up to the $3 poverty line and for every above-mentioned type of construction for the time spans.

Table 1.7 shows the results of such a dominance test. Obviously, according to our proposed methodology, no significant ordering of poverty dynamics can be established for the time span 1998-2002. This is a very interesting result given the large number of 1’s in Table 1.7. Suppose the objective is to assess chronic poverty for the time span 1998-2002. Using the $2 poverty line and comparing the time spans $[1998;1999;2000]$ and $[2000;2001;2002]$ – which might be judged a reasonable comparison at first glance – one would conclude that chronic poverty has fallen. However, taking the time spans $[1998;1999]$ and $[2001;2002]$ shows instead that no conclusion can be drawn. Hence, the poverty ordering depends not only on the chosen poverty line but also on the way the time spans are constructed.

### 1.4 Discussion

In this paper, we presented a concept allowing to undertake multiperiod poverty comparisons over time and space without arbitrarily aggregating income over various years. Inspired by the multidimensional stochastic dominance methodology...
Table 1.7: Poverty in Peru – Difference in Dominance Surfaces for Several Construction Modes of Time Spans

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<thead>
<tr>
<th>Income</th>
<th>[98;00] vs. [00;02]</th>
<th>[98;99] vs. [99;02]</th>
<th>[98;01] vs. [00;01;02]</th>
<th>[98;99;00] vs. [99;00;01]</th>
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Income: Household income per capita per day in US$; 1 indicates that the earlier surface was significantly above later surface, −1 indicates the opposite, 0 indicates no significant difference. Significance level: 5%. Source: Authors’ calculations based on ENAHO

elaborated by Duclos et al. (2006b), we created n-period income surfaces for different time spans. These surfaces were then ordered using dominance tests. Once dominance is established, the poverty ordering is robust to a wide range of poverty indices, to a wide range of poverty lines, and to a wide range of aggregation procedures. Furthermore, we extended our framework to the measurement of relative poverty.

To illustrate our methodology, we compared poverty across time spans in Peru and between Peru and Indonesia. Furthermore, we highlighted some general problems of dynamic poverty comparisons, i.e. how time spans should be constructed, namely which period should be the end of the first and the beginning of the second time span and how many periods should constitute a time span? We dealt with these questions by applying robustness test with respect to various of these possibilities.

However, the approach suggested and the ideas developed in this paper also have their limitations. The most important of these is certainly that all results are based on a sample of expenditures declared by households and that these declarations are generally affected by measurement error, which affects the bivariate distribution $F(y_1, y_2)$ (and n-variate distribution) much more than the univariate distribution $F(y)$. In fact, many empirical studies show that measurement error is such that the extent of $\beta$-convergence over time is overestimated (see Bound et al. (2001); Breen and Moisio (2004); Grimm (2007)). For our case, this would imply that multiperiod poverty is underestimated. In the absence of information on ‘true income’ or any instruments, there is not much that can be done about this, but it should be kept in mind when interpreting our results. However, the problem is obviously not specific to our approach but inherent in most approaches to the analysis of poverty dynamics.