III. A New Keynesian model with endogenous capital with adjustment costs

One of the main findings concerning rule-based monetary policy design in Chapter II has been that the assessment of different rule specifications in terms of the criteria presented in Section 2 (determinacy of rational-expectations equilibrium and response to shocks), as well as the evaluation of the relevance of the Taylor principle for fulfilling these criteria are to a great extent dependent on the model used and the inclusion of a supply-side transmission channel in particular.

Therefore, in this chapter I focus on deriving a New Keynesian model with endogenous capital and adjustment costs and proceed with examining the system’s determinacy properties under different rule specifications. In terms of the general applicability of the Taylor principle, the findings are non-trivial. In the model with endogenous capital an inflation response coefficient greater than unity is not sufficient *per se* for ensuring a unique rational-expectations equilibrium. Apart from a small value interval of the inflation response coefficient above unity, implying a moderately strong policy reaction, determinacy under an “active” rule requires some degree of output gap response. Moreover, even within the inflation coefficient interval yielding determinacy under a sole inflation target in the rule, adding an output gap term is still associated with a unique equilibrium. In terms of “passive” rules (with an inflation response coefficient below one), the Taylor principle is only partially valid as well. Indeed, if inflation is the only target variable entering the rule, a smaller than one-on-one nominal interest rate response to inflation deviations yields indeterminacy of rational-expectations equilibrium. However, introducing a sufficiently large output gap response can lead to a unique equilibrium even under a passive rule.

Thus, modelling endogenous capital provides important new insights and an extension to the baseline formulation of the Taylor principle, namely by adding the requirement of introducing an output gap response in order to guarantee uniqueness of the system’s rational-expectations equilibrium.

The chapter is organised as follows. In Sections 1 and 2, I give an overview to the baseline New Keynesian framework and possible approaches to modelling capital and investment. The model with endogenous capital and adjustment costs, which is the basis for the determinacy and impulse response analysis to follow in the next chapters, is presented in Section 3. In this section, I first examine the households’ optimisation problem. The resulting first-order conditions describe the aggregate demand side of the model. Then I present the producers’ optimisation and derive the aggregate supply curve and the real-wage equation and add an interest rate rule to the system. Dynamics of the whole economy are

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73 In fact, as it is evident in the analysis in Section 4, the output gap response required for determinacy increases as the inflation response coefficient decreases.
thus fully characterised by combining equilibrium conditions from both the demand and the supply side. As a next step, in Section 4 these log-linearised equilibrium conditions are used to study the determinacy property of interest rate rules. Then, in Section 4 I present the responses generated under different interest-rate-rule specifications to three types of shocks. Section 5 provides a preliminary summary of results.

1. The New Keynesian framework: an overview

Over the past decade numerous examples of small-scale monetary business cycle optimising models featuring nominal rigidities have appeared in the literature. They are generally known as New Keynesian models. Both their theoretical appeal as micro-founded models, and their ability to explain the short-run effects of monetary policy, have contributed to their popularity among researchers. Taylor (2000) describes New Keynesian models as “…dynamic, stochastic, economy wide models with forward-looking behaviour and some rigidities that make them useful for policy evaluation”. This kind of models are also sometimes referred to as “Dynamic New Keynesian” (Bernanke et al. (1998)) or New Neoclassical Synthesis (Goodfriend and King (1997)).

New Keynesian models typically integrate standard Keynesian elements (imperfect competition, nominal rigidities in price- and wage-setting) into a dynamic general equilibrium framework with rational expectations of market participants. One substantial improvement in recent research in comparison to the traditional Keynesian framework consists in stronger theoretical and microeconomic foundations. Behavioural functions for aggregate variables are derived from optimal individual behavior of households and firms with simultaneous clearing of all markets. Thus, these models are an appropriate tool for analysing the connection between interest rates, inflation and the business cycle, as well as for comparing the impact of alternative monetary policies.

An important feature of New Keynesian models is the inclusion of rational expectations of market participants. Muth (1961) first formulated the rational expectations hypothesis, which requires that the subjective expectation of economic actors (households and firms) regarding a particular variable be equal to the objective expectation for that variable conditional on the information set available. In the following decade the idea has been further developed, among others, by Lucas (1972, 1973), Sargent (1973), Sargent and Wallace (1975, 1976) and Barro (1976).

Another essential characteristic of New Keynesian models concerns the nature of inflation dynamics under monopolistic competition reflected in the New Keynesian Phillips Curve. Under the widely adopted staggered price specifica-

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74 In the original paper, Muth suggested that “...expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory” (1961, p. 316).
tion as in Calvo (1983), inflation has forward-looking character as a result of the assumption that firms face constraints on the frequency of price-adjustment. This means that previously adjusted prices are likely to remain effective for longer than one period, i.e. current price-setting decisions (and therefore current inflation) are based on expectations about future cost and demand developments. Another determinant of inflation dynamics are mark-up variations (or real marginal cost variations) that arise from the monopolistic firms’ repeated attempts to adjust actual to desired mark-ups. Roberts (1998) suggests that the aggregate supply equation fits better to empirical data if the rational expectations assumption is replaced by a partially backward-looking model of expected inflation. The output gap is an endogenous variable in the New Keynesian models, related to the ex ante real interest rate and expected output gap in the aggregate demand relation. Frequently the variable enters as an inflation fluctuations determinant the aggregate supply relation and as a policy target the central bank reaction function.

The empirical relevance of New Keynesian models has often been criticised. The dependence of optimising New Keynesian models on a forward-looking decision making process limits their capacity to capture some of the business cycle regularities observed in the data. For instance, most optimising models are not very successful in replicating the delay in the responses of output and inflation to a monetary shock. In particular, an optimising model should explain why, rather than immediately, responses of both output and inflation to a monetary impulse reach their maximal impact several quarters after the shock. This phenomenon has been widely investigated in recent papers using optimising models incorporating frictions in price-setting and/or wage-setting, e.g. in Chari et al. (2000), Christiano et al. (2001) and Giannoni and Woodford (2003).

The canonical New Keynesian model, as well as most of its standard generalisations, abstracts from investment in order to maintain simplicity. One possible explanation is the emphasis on short-run analysis of macroeconomic stabilisation processes that allows abstracting from long-term capital accumulation implications. Moreover, the exclusion of capital is often justified on the grounds that the capital stock is not characterised by substantial volatility at business cycle frequencies and empirically there is a very small correlation between capital

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75 As mentioned in the last paragraph, the output gap is sometimes substituted at the place of real marginal cost as inflation determinant. In fact, the Phillips curve relation derived from staggered price-setting as in Calvo (1983) involves the deviation of real marginal cost from its steady-state value. As Galí and Gertler (1999) and Clarida et al. (1999) show, certain assumptions about technology, preferences and the labour market structure can be made that infer a proportionate relation between real marginal cost and the output gap.

76 This view has been expressed by McCallum and Nelson (1999b). Examples of New Keynesian models with constant (exogenous) capital and investment include, among others, Kerr and King (1996), Bernanke and Woodford (1997) and Clarida et al. (2000).
and aggregate output measures (see McCallum and Nelson (1999a)). Difficulties
with empirical measures of the capital stock also discourage developing models
that involve capital accumulation.

However, Dennis (2004) argues that abstracting from investment may imply
that an important shock propagation mechanism that may have important impli-
cations for the design and implementation of an optimal monetary policy is
omitted. Woodford (2003) provides a further critique on models with constant
capital: “while this has kept our analysis of the effects of interest rates on aggre-
gate demand quite simple, one may doubt the accuracy of the conclusions ob-
tained, given the obvious importance of variations in investment spending both
in business fluctuations and in the transmission mechanism for monetary policy
in particular.” Casares and McCallum (2006) provide a further argument for the
inclusion of endogenous capital and investment, as it enables not only studying
issues relating to capital formation and growth, but also provides an endogenous
explanation for the empirically observed contrasting variability of consumption
and investment spending.

When introducing investment within a New Keynesian framework, a crucial
choice to make involves modelling the speed of capital stock adjustment in the
occurrence of shocks. The case when capital adjusts relatively fast can be repre-
sented by modelling endogenous investment with an economy-wide rental mar-
et77 as in Hairault and Portier (1993), Kimball (1995), Yun (1996), King and
Watson (1996), King and Wolman (1996) and Chari et al. (2000). A further op-
tion consists in introducing a certain degree of inertia in capital accumulation in
the model, which, as an additional advantage, seems to match better empirical
data on capital stock dynamics. This can be achieved by introducing assump-
tions that prevent the capital stock from immediately responding to shocks. As
already mentioned in Section 1, three possible assumptions about investment
and the capital stock could generate an inertial response on part of capital: capi-
tal accumulation adjustment costs, a time-to-build requirement and firm-specific
capital. As shown by Casares and McCallum (2006), an appropriate possibility
to endogenise investment in a less complex manner is to incorporate endogenous
investment with capital adjustment costs under sluggish price adjustment in a
dynamic model of the IS-LM type with optimising behaviour78. This is approach
chosen in the next section for deriving the New Keynesian model that is later
used for examining the determinacy and shock response properties of different

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77 My results obtained under such a specification without adjustment costs (not included
here) confirm the intuition that variable responses to shocks in such a case are unrealisti-
cally large. This makes the choice of such a modelling option quite unappealing for
policy analysis.

78 Such models have been used by Woodford (1995), Kerr and King (1996), Rotemberg
interest rate rule specifications in Section 4 (Chapter III) and Sections 2 and 3 (Chapter IV).

2. Modelling capital and investment

Ever since Keynes (1936) and Hicks (1939), the role of investment dynamics in business cycle fluctuations has been highlighted in macroeconomic analysis\(^\text{79}\). Some prominent models include the neoclassical theory of investment, the accelerator model and Tobin’s q theory.

The neoclassical theory of investment\(^\text{80}\) uses the firm's production function and IS curve to derive demand for capital. Based on the IS curve and production function a relation between a firm's cash flow and its contemporaneous stock of fixed capital (plant and equipment) is obtained. The firm's demand for fixed capital is then set at a level that equates the marginal profit of capital with the user cost of capital\(^\text{81}\). The neoclassical theory of investment postulates that a firm's demand for capital is positively related to the firm's level of output and negatively related to the user cost of capital. Among the more significant empirical results reported by Jorgenson and associates\(^\text{82}\) are that (i) investment demand is highly responsive to changes in relative prices, including policy variables such as the interest rate and taxes; (ii) the lag to the investment response to changes in its determinants is relatively long (on the average, about eight to nine quarters) and there is no response in the first few quarters and (iii) the distributed lag structure of investment behavior is bell-shaped; gross investment initially rises at an increasing rate and then increases at a decreasing rate as long-run equilibrium is approached. A more simplistic neoclassical framework, in which the demand for capital is still determined by the output level, but not by the user cost of capital, is the accelerator model of investment\(^\text{83}\).

Both the Jorgenson and the accelerator model treat steady-state capital stock as the desired level of capital stock and then impose an adjustment mechanism of actual capital stock towards its desired (steady-state) level\(^\text{84}\). Rather than de-
riving a particular dynamic capital adjustment mechanism in the firm’s optimisation problem (for instance, based on an adjustment cost function), these models assume that there exists an exogenous mechanism that determines the speed of the adjustment of actual to desired capital stock.

An alternative approach is to incorporate adjustment costs and the price of investment goods directly into the maximization problem and then to derive the optimal rate of investment in each period. The q theory of investment as in Tobin (1969) explicitly derives the dynamic response of investment to permanent and temporary (as well as anticipated and unanticipated) changes in the underlying determinants. Tobin’s q theory of investment builds on a baseline Keynesian idea that the investment decision is influenced by the market value of capital relative to the cost of acquiring additional capital. In case an additional unit of capital would increase the market value of the firm by more than the cost of acquiring the capital and putting it in place, then an optimizing firm should decide in favour of investment. Although the original version of the q theory did not explicitly model the firms’ production function and demand curve, it is possible to start with the IS curve and the production function and then derive the q theory as the result of intertemporal maximisation by firms.

Since the beginning of the 1990s, numerous contributions have aimed at incorporating the findings of neoclassical investment theory on the lagged character of capital and investment responses within a forward-looking New Keynesian framework with dynamic adjustment costs, micro-foundation and nominal price and wage rigidities. As Casares (2006) argues, introducing slow capital adjustment over time helps to replicate the delay in the responses of output and inflation to a shock, observed in the data. Several options to model the slow adjustment of the capital stock have been discussed in the recent literature: dynamic capital adjustment costs, time-to-build assumption, restricted capital stock mobility across sectors or firms (firm-specific capital).

The approach of assuming capital adjustment costs reflects the fact that modifying the scale of capital services in a firm generates disruption costs during installation of any new or replacement capital. As a result, costly learning must be incurred as the structure of production may have been changed. Installing new equipment or structures frequently implies delivery lags and time to install/build. The irreversibility of many projects caused by a lack of secondary

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85 Tobin (1969) defined q to be the ratio of the market value of a firm to the capital replacement cost of the firm. This ratio measures the value of fixed capital relative to its cost. A high ratio enhances incentive to acquire the capital and therefore the rate of investment. As the value of the firm is measured using data from financial markets, the link between asset markets and investment expenditure is quite straightforward.
markets for capital goods acts as another form of adjustment cost. Among others, Eisner and Strotz (1963), Lucas and Prescott (1971), and Hayashi (1982) have incorporated the assumption that the process of installing capital goods requires the use of resources. The baseline model in the investment literature has been a standard neoclassical model with convex (often approximated to be quadratic) costs of adjustment. Caballero (1999) shows that this model has not performed well at an aggregate level; besides, some industry studies suggest that both convex and non-convex adjustment costs can be observed in practice. That is why an alternative approach to the standard convex cost model has been advocated recently by Abel and Eberly (1994, 1996), Caballero et al. (1995), and Cooper et al. (1999), emphasizing that non-convexities and irreversibility play an important role in the investment process. Paez-Farrell (2003) points out that the realistic modeling of adjustment costs is hindered by the fact that they are difficult to quantify.

Still, Caballero and Engel (1999), Thomas (2002) and Cooper and Haltiwanger (2006) find that, despite performing poorly at the plant level, a model with convex costs fits the aggregate data reasonably well. Therefore, despite the inaccuracy problems that are incurred by approximating capital adjustment costs by convex costs, this modeling choice is appealing due to it relative simplicity. The above deficiencies notwithstanding, models with endogenous capital and convex capital adjustment costs still perform better than the ones with endogenous capital only. This is confirmed by Casares and McCallum (2006) who argue that models with endogenous capital/investment choices but no adjustment costs imply highly unrealistic responses to monetary-policy shocks under the assumption of sticky prices and wages.

A second option of modelling the slow adjustment of capital over time, consistent with the empirical data, is to introduce lags to the investment process, for instance by adopting a time-to-build assumption. This approach can be traced back to Kydland and Prescott (1982) who construct a one-sector optimal growth model with persistent technology shocks. In order to design a more complex propagation mechanism for such shocks and to incorporate the co-movement of output and investment over time in a more realistic manner, they introduce a time-to-build lag in capital stock adjustment and achieve a good match of the

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86 McDonald and Siegel (1986) argue that if investment is irreversible (e.g. building a plant), the optimal decision of firms might be to forego some investments whose present value exceeds their cost. Then the correct calculation involves comparing the value of investing today with the (present) value of investing at all possible times in the future.

87 A detailed overview of convex adjustment cost models and numerous references to the motivation and results of the literature are provided by Hamermesh and Pfann (1996).

88 See, for example, Holt et al. (1960) and Peck (1974).

89 However, they also find that convex cost models tend not to track investment at turning points well.
simulation results to quarterly U.S. postwar business cycle data. In a similar line, Taylor (1983) calculates optimal investment cost-reducing policy rules to offset investment fluctuations in a model with dynamic investment optimisation at firms’ level. In the Taylor (1983) model, investment dynamics are generated by heterogeneous gestation lags between the start and completion of investment projects rather than by introducing capital adjustment costs. A more recent paper by Paez-Farrell (2003) highlights that, by introducing the time-to-build assumption, two key difficulties surrounding basic sticky price models can be overcome, namely, the high volatility of the variables and the immediate impact of monetary policy shocks.

Other recent contributions study the responses to monetary shocks under a time-to-build assumption in a more elaborated framework. Casares (2006) examines the effects of a monetary policy shock across two alternative time-to-build specifications and a model with no time-to-build feature and finds that the inclusion of the time-to-build assumption matters for the response of investment, output and inflation to an interest rate shock. The multiple-period time-to-build model reports realistic (u-shaped) responses of the above mentioned variables, while the models with no or a one-period time-to-build structure fail to replicate empirical developments. Edge (2007) also develops a model with several time-to-build requirements for household capital accumulation but without imposing a time-to-build constraint on the demand for capital of the production sector. His findings provide further arguments in favour of the introduction of a time-to-build feature in order to replicate the adjustment pattern as a result of a monetary shock, evident in the data. He concludes that the assumptions that capital takes time to build and to plan, and that investment plans are costly to change once they are underway act to reduce the response of investment following a monetary shock.

On the other hand, Rouwenhorst (1991) criticizes the conclusion that the time-to-build feature can actually be crucial to the explanation of business cycles. By comparing models with and without this assumption, he finds that time-to-build can introduce cyclical behaviour to the dynamic adjustment of consumption, output and investment only as a result of the stochastic processes for the shocks that hit the economy and concludes: “Time to build, by itself, contains only relatively weak material-propagation mechanisms for transferring real shocks, in terms of effects on output, labour input, and consumption. For persistent deviations of output and investment to occur in the neoclassical model, it is required that the time series of shocks that hit the economy behave very much like the fluctuations which the model aims to explain... Thus the paper concludes that time to build does not seem to be central to the explanation of business cycles” (p. 242). A more general critique is provided by Blanchard and Fischer (1989) who argue that the time-to-build approach becomes much less
tractable once no capital rental markets are assumed. In this case, when making an investment decision, the firm must also assess the probability that the conditions could change in the meantime and cause it to want to stop the project.

The notion that capital can be firm-specific in the Blanchard and Fischer (1989) critique above has been further highlighted by more recent research. Woodford (2005) finds that the assumption of a competitive rental market for capital services in which at every point in time the shadow cost of capital is equal across firms and sectors is unrealistic, as it could imply that a substantial part of the aggregate capital stock shifts each period from low-demand to high-demand producers. He also argues that the rental market feature has non-trivial implications for the evolution of marginal cost at the firm level and hence for price-setting and inflation dynamics. He finds that, in terms of price stickiness, incorporating a rental market for capital results in a substantial exaggeration of the infrequency of price adjustment, while assuming exogenous capital instead results in a smaller underestimate. The insight that firm-specific capital leads to a lower estimate of the degree of price stickiness has been confirmed in a framework with constant firm-specific capital by Sbordone (1998) and Galí et al. (2001). Later Eichenbaum and Fisher (2004), as well as Altig et al. (2005), building on the research by Woodford have analysed the consequences of endogenous firm-specific capital for the estimated frequency of price adjustment in empirical versions of the New Keynesian Phillips curve in order to allow a closer fit to U.S. empirical data. Woodford (2005) highlights that the introduction of firm-specific capital solves the “micro/macro” conflict, i.e. the differing parameter values required to explain the co-movement between aggregate inflation and aggregate output (as estimated by Phillips curve time series) and the parameter values consistent with microeconomic observations. He argues that the discrepancy between the frequency of price adjustment required to explain the aggregate inflation/output co-movements and the one that is suggested by microeconomic data disappears once certain more realistic assumptions are introduced to the model (such as firm-specific capital, industry-specific labour markets, intermediate inputs entering production, non-constant elasticity of substitution among differentiated consumption and investment goods).

As far as aggregate inflation dynamics is concerned, Sveen and Weinke (2004), as well as Woodford (2005) find that the introduction of firm-specific endogenous capital does not alter the results under constant (exogenous) capital stock. Woodford (2005) argues that the same form of equilibrium relation between inflation dynamics and the evolution of average real marginal cost can be derived under firm-specific capital. Moreover, he finds that the relation between the slope of the Phillips curve and the frequency of price adjustment that can be

90 See Chapter 6, p. 297.
91 The first version of Woodford (2005) with form-specific endogenous capital dates back to 2003 (see Chapter 5 in “Interest and Prices”).
derived under the simpler assumption of an exogenously given capital stock for each firm seems to be fairly accurate as an approximation to the correct relation in the case of firm-specific capital with an empirically realistic size of investment adjustment costs. In addition, Sveen and Weinke (2004) provide an elaborate explanation of the similarity in terms of inflation dynamics resulting from a monetary shock between the model with endogenous firm-specific capital and the baseline version with exogenous capital stock. Under firm-specific capital, there are two opposite effects on marginal costs. In the first place, the additional output generated by investment demand raises marginal costs; on the other hand, the additionally accumulated capital stock enhances the economy’s productive capacity and thus reduces marginal costs. In terms of the output response, Sveen and Weinke (2004) find that, both on impact and during the transition period, the response of output is higher under the assumption of firm-specific capital than in a model with exogenous capital.

3. The model with endogenous capital and adjustment costs

3.1. Household utility function and optimality conditions

Under the assumption of identical capital and non-capital wealth in the initial period, complete financial markets and homogenous factor prices, all households make identical consumption, investment and factor supply decisions. Thus the household sector of the economy can be characterised by a representative household that is both a consumer and an owner of production factors. It seeks to optimise the intertemporal utility function

\[ E \sum_{j=0}^{\infty} \beta^j U(c_{ij}, l_{ij}, m_{ij}, v_{ij}, \xi_{ij}) \]

(3.1)

where \( c_i, l_i, \) and \( m_i \) are consumption, leisure and the stock of real money balances, \( E \) is the expectation operator conditional on information available at \( t \) and \( \beta = (1 + \rho_U)^{-1}, \rho_U > 0 \) is the household’s discount factor. The terms \( v_i \) and \( \xi_i \) denote two preference shocks, affecting consumption and the household’s holdings of the economy’s medium of exchange, respectively. On the supply side, the representative household is engaged in producing specialised output. The household’s production is described by a homogenous Cobb-Douglas production function of degree 1:

\[ f(A_i p^d_i, k_i) = (A_i p^d_i)^{1 - \alpha} k_i^{\alpha}, \quad 0 < \alpha p^d < 1 \]

(3.2)

92 Here the plural form used reflects the fact that, under firm-specific capital, due to the absence of an economy-wide rental market, there exist as many quantitatively differing marginal costs as there are individual producers in the economy.
where $\alpha_{pf}$ is the elasticity of substitution between capital and labour, $A_t$ is a (labour-augmenting) technology shock, $n^d_t$ and $k_t$ denote the household’s labour demand and capital stock at time $t$. Sales of the household’s specialised output are constrained by the demand function

$$Y_t^A \left( \frac{P_t}{P_t^A} \right)^{-\theta},$$

(3.3)

where $Y_t^A$, $P_t$ and $P_t^A$ denote aggregate demand, the price of the household’s product and the aggregate price level. The elasticity of substitution across differentiated consumption goods is denoted by $\theta$. In addition, the representative household supplies differentiated labour services on the monopolistically competitive labour market. The quantity demanded is given by

$$n_t^A \left( \frac{W_t}{W^A_t} \right)^{-\theta_L},$$

(3.4)

where aggregate labour is denoted by $n_t^A$, $W_t$ is the household’s nominal wage, $W^A_t$ is the aggregate nominal wage and $\theta_L$ is the elasticity of substitution for the differentiated labour services. At the same time, the representative household buys time units of a Dixit-Stiglitz composite labour input at the real wage rate $\omega_t = W_t^A / P_t^A$. The household’s budget constraint in $t$ is given by

$$Y_t^A \left( \frac{P_t}{P_t^A} \right)^{-\theta} - \tau_t - \omega_t \left( n_t^d - n_t^A \left( \frac{W_t}{W^A_t} \right)^{\theta_L} \right) = c_t + k_{r+1} - (1-\delta)k_t + m_t - (1+\pi_t)^{-1} m_{r+1} + (1+r_t)^{-1} b_{r+1} - b_t,$$

(3.5)

where $\pi_t = (P_t^A / P_{t-1}^A) - 1$ is the inflation rate, $b_{r+1}$ denotes one-period government bonds purchased in $t$ with real price $(1+r_t)^{-1}$, $\tau_t$ stands for lump-sum taxes (net of transfers) paid by the household and $k_t$ is the capital stock and $\delta$ is the capital depreciation rate. Next, I introduce two equilibrium conditions for the representative household’s production and labour supply. Production is equal to the quantity that is demanded as in

$$f(A, n_t^d, k_t) = Y_t^A \left( \frac{P_t}{P_t^A} \right)^{-\theta}.$$

(3.6)

Labour supply is determined by the labour demand in the monopolistic competition labour market and is equal to

$$n_t^A \left( \frac{W_t}{W^A_t} \right)^{-\theta_L} + l_t = 1.$$

(3.7)

93 If constant labour input is assumed, (3.5) is reduced to

$$Y_t^A \left( \frac{P_t}{P_t^A} \right)^{-\theta} - \tau_t (n_t - 1) = c_t + k_{r+1} - (1-\delta)k_t + m_t - (1+\pi_t)^{-1} m_{r+1} + (1+r_t)^{-1} b_{r+1} - b_t.$$
The household’s optimality conditions consist of (3.5) - (3.7), together with

\[ \begin{align*}
U_i(c_i, m_i, l_i, \nu_i, \zeta_i) - \lambda_i &= 0 \quad \text{(3.8)} \\
U_2(c_i, m_i, l_i, \nu_i, \zeta_i) - \lambda_i + \beta E_{t+1} \left[ \lambda_{t+1} (1 + \pi_{t+1})^{-1} \right] &= 0 \quad \text{(3.9)} \\
U_3(c_i, m_i, l_i, \nu_i, \zeta_i) - \zeta_i &= 0 \quad \text{(3.10)} \\
-\lambda_i \omega_i + \vartheta_i A_i f_i(A_i n_i^d, k_i) &= 0 \quad \text{(3.11)} \\
-\lambda_i + \beta E_{t+1} \lambda_{t+1} (1 - \delta) + \beta E_{t+1} \left[ \vartheta_{t+1} f_2(A_i n_{t+1}^d, k_{t+1}) \right] &= 0 \quad \text{(3.12)} \\
-\lambda_i (1 + r_i)^{-1} + \beta E_{t+1} \lambda_{t+1} &= 0 \quad \text{(3.13)} \\
\lambda_i Y_i^A (1 - \theta) P_i^{-\theta} / (P_i^d)^{1-\theta} + \vartheta_i Y_i^A \theta P_i^{-(\theta+1)} / (P_i^d)^{-\theta} &= 0 \quad \text{(3.14)} \\
\lambda_i n_i^d (1 - \theta) i_{t+1} W_i^{-\theta} / (W_i^d)^{1-\theta} + \omega_i n_i^d \theta_i W_i^{-(\theta+1)} / (W_i^d)^{-\theta} &= 0 \quad \text{(3.15)}
\end{align*} \]

whereby \( \lambda_i, \vartheta_i \) and \( \omega_i \) are the Lagrange multipliers to (3.5), (3.6) and (3.7) respectively. The marginal product of labour and capital are denoted by \( n_i^d \) and \( n_i^k \). Equations (3.5)- (3.15) determine the paths of \( c_i, m_i, l_i, n_i^d, k_{t+1}, b_{t+1}, P_i, W_i, \lambda_i, \vartheta_i \) and \( \omega_i \), given \( \pi_i, \omega_i, r_i, P_i^d, W_i^d \) and \( tx_i \). For general equilibrium, there are two market clearing conditions:

\[ \begin{align*}
n_i^d &= \sum n_i^d \quad \text{(3.16)} \\
m_i &= \frac{M_i}{P_i^d} \quad \text{(3.17)}
\end{align*} \]

the identity

\[ \pi_i = \frac{P_i^d}{P_i^{d-1}} - 1 \quad \text{(3.18)} \]

and the government’s budget constraint

\[ g_i - tx_i = m_i + (1 + \pi_i)^{-1} m_{t+1} + (1 + r_i)^{-1} b_{t+1} - b_i \quad \text{(3.19)} \]

whereby \( M_i \) and \( g_i \) denote the nominal money supply and government consumption of goods and services per household, while \( r_i \) is the real interest rate. Without nominal rigidities, \( P_i = P_i^d \) and \( W_i = W_i^d \). Altogether, equations (3.5)-(3.19) determine the paths of \( c, m, l, n_i^d, n_i^k, k, b, P, W, \lambda, \vartheta, \omega, r \) and \( \pi \) in response to the exogenous paths of \( M_i, g_i \) and \( tx_i \). Alternatively, it can be assumed that the government sets the path of \( b_i \) instead of \( g_i \) or \( tx_i \). By analogy, the central bank can implement monetary policy by determining the nominal interest rate \( i_t \) instead of using \( M_i \) as an instrument. The latter option will be pursued further in Subsection 3.5.

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94 Here \( f_i(.) \) denotes the partial derivative of the function \( f(.) \) with respect to its \( i \)-th argument.
3.2. The “IS sector”

Having presented the model’s micro-foundations in Subsection 3.1, some of the “IS sector” relations will be derived in this subsection, including the consumption equation and the overall resource constraint. The equations relating to capital and investment will be presented in Subsection 3.3. In order to derive the consumption relation for the “IS block”, (3.8) and (3.13) can be combined to yield

\[ U_1(c_i, m_i, l_i, v_i, \xi) = \beta E_t[U_1(c_{t+1}, m_{t+1}, l_{t+1}, v_{t+1}, \xi_{t+1})(1 + r)] \]  

(3.20)

Then, combining (3.8), (3.9) and (3.13) yields

\[ U_2(c_i, m_i, l_i, v_i, \xi) = U_1(c_i, m_i, l_i, v_i, \xi) \frac{i_t}{1 + i_t} \]  

(3.21)

with \( 1 + i_t = (1 + r_t)(1 + E_t \pi_{t+1}) \).

The period utility function is approximated by \(^95\)

\[ U(c_i, m_i, l_i, v_i, \xi) = \gamma \exp(v_i) \frac{c_i^{-\sigma}}{1 - \sigma} + (1 - \gamma) \exp(\xi) \frac{m_i^{1-\gamma}}{(1 - \gamma)} + \Lambda \frac{i_t^{-\tau}}{1 - \tau} \]  

(3.22)

or

\[ \exp(v_i) c_i^{-\sigma} = E_t \left[ \exp(v_{t+1}) c_{t+1}^{-\sigma} \right] \beta(1 + r) \]  

(3.23)

with \( U_1(c_i, m_i, l_i, v_i, \xi) = \gamma \exp(v_i) c_i^{-\sigma} \), \( U_2(c_i, m_i, l_i, v_i, \xi) = (1 - \gamma) \exp(\xi) m_i^{1-\gamma} \), \( 0 < \gamma < 1 \) and \( \sigma, \gamma, \tau, \Lambda > 0 \). Equation (3.23) can be transformed to yield

\[ \hat{c}_t = E_t \hat{c}_{t+1} - \sigma^{-1}(r_t - \bar{r}) - \sigma^{-1}(E_t \nu_{t+1} - \nu_t) \]  

(3.24)

whereby the “hat” variables denote logarithmic fractional deviations from the respective steady-state values, i.e. \( \hat{c}_t = \log \left( c_t / \bar{c} \right) \). Assuming that the consumption preference shock follows the AR (1) process \( \nu_t = \rho \nu_{t-1} + \varepsilon_t \), and also substituting \( r_t = i_t - E_t \pi_{t+1} \) in (3.24) yields the following consumption equation:

\[ \hat{c}_t = E_t \hat{c}_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - \bar{r}) + \sigma^{-1}(1 - \rho) \nu_t \]  

(3.25)

Next, the log-linear approximation to the overall resource constraint can be written as

\[ \hat{y}_t = \omega_c \hat{c}_t + \omega_g \hat{g}_t + \omega_{inv} \hat{inv}_t \]  

(3.26)

where the value of the coefficient \( \omega_{inv} \) depends on the share of investment in total output in steady-state, i.e. \( \omega_{inv} = \overline{inv}(1 + f(\overline{inv}, \overline{k})) / \overline{y} \). By analogy, the steady-state shares in output of consumption and government expenditure are given by \( \omega_c = \overline{c} / \overline{y} \) and \( \omega_g = \overline{g} / \overline{y} \).

---

\(^95\) The period utility function is separable in terms of consumption and real money balances.
3.3. Capital accumulation adjustment costs

In order to specify realistic movements of capital and investment, it is necessary to add investment adjustments costs to the model. As already discussed in Section 1, some plausible ways of achieving this would include adopting the assumption that capital investment (or capital good “installation”) creates certain costs or adding exogenous “time to build” constraints. Here I choose to endogenise the sluggishness of capital stock adjustments and adopt the adjustment-cost specification as in Hayashi (1982) and Casares and McCallum (2006).

Gross investment is given by:

\[ inv_t = k_{t+1} - (1 - \delta)k_t \]  
(3.27)

Adjustment costs take the form

\[ C(inv_t, k_t) = inv_t f \left( \frac{inv_t}{k_t} \right) \]  
(3.28)

where the unit capital installation cost depends on the investment-capital ratio \( \frac{inv_t}{k_t} \) according to

\[ \frac{C(inv_t, k_t)}{inv_t} = f \left( \frac{inv_t}{k_t} \right), \]  
(3.28')

with \( f'(\cdot) > 0 \) and \( 2f''(\cdot) + \delta f'''(\cdot) > 0 \). Within this specification, total adjustment cost of investment in \( t \) varies with both \( inv_t \) and \( k_t \). The production function with adjustment costs \( f(Ainv^\delta, k_t) - C(inv_t, k_t) \) is assumed to be homogeneous of degree 1, implying constant returns to scale \(^96\), i.e. the size of the plant has no influence on the steady-state ratio of adjustment cost to output \( \frac{C(inv, k)}{\bar{y}} \). If the functional form \( f \left( \frac{inv_t}{k_t} \right) = \Theta_t \left( \frac{inv_t}{k_t} \right)^{\eta_2} \) is assumed, total adjustment costs are given by

\[ C(inv_t, k_t) = \Theta_t \frac{inv_t^{\eta_2 + 1}}{k_t^{\eta_2}}. \]  
(3.29)

If adjustment costs for investment are introduced, the household’s budget constraint (3.5) from Subsection 3.1 becomes

\(^96\) Another possibility is to specify adjustment costs as in Abel (1983), where total adjustment cost of investment in \( t \) varies only with gross investment according to \( C(inv_t) = \xi inv_t^\eta \) where the scale parameter \( \xi > 0 \) represents adjustment costs and \( \eta > 1 \) is the elasticity of total adjustment costs with respect to investment. Then, the values of the parameters \( \xi \) and \( \eta \) imply increasing marginal adjustment costs as result of a rise in gross investment. Under a homogeneous production function of degree 1, subtracting adjustment costs from the production function \( f(Ainv^\delta, k_t) - C(inv_t) \) implies decreasing returns to scale.
After introducing capital adjustment costs of the form (3.29), the capital stock first-order optimality condition in $t+1$ takes the form

$$
-\lambda_t [1 + C_t (inv_t, k_t)] + \beta E_t \lambda_{t+1} [1 - \delta - C_{t+1} (inv_{t+1}, k_{t+1})] + \beta E_t \left[ p_{t+1} \right] = 0.
$$

Marginal adjustment costs in $t$ and $t+1$ are given by $C_t = C_t (inv_t, k_t)$ and $C_{t+1} = C_{t+1} (inv_{t+1}, k_{t+1})$. After substituting for next period’s real marginal cost $rmc_{t+1} = r_{t+1} / mpl_{t+1}$ from (3.11), (3.12’’) takes the form

$$
\lambda_t (1 + C_t) = \beta E_t \lambda_{t+1} (1 - \delta - C_{t+1}) + \beta E_t \lambda_{t+1} (rmc_{t+1} mpk_{t+1}) .
$$

Substituting $\beta E_t \lambda_{t+1} = \lambda_t (1 + r) - 1$ from (3.13) into (3.12’’) yields

$$
1 + r_t = \frac{1 - \delta - E_t C_{t+1} + E_t (rmc_{t+1} mpk_{t+1})}{1 + C_t},
$$

where $1 + r_t$ is the return on the financial asset (the opportunity cost for investment). The right-hand side denotes the expected net marginal return on investment in the real asset. After substituting $C_t$ and $E_t C_{t+1}$, the marginal cost in $t$ and the marginal cost expected in $t+1$, from (3.29), (3.30) becomes

$$
1 - \delta + (1 - \delta) \Theta_1 (\Theta_2 + 1) \left( \frac{E_{inv_{t+1}}}{k_{t+1}} \right)^{\Theta_2} + \Theta_1 \Theta_2 \left( \frac{E_{inv_{t+1}}}{k_{t+1}} \right)^{\Theta_2 + 1} + E_t (rmc_{t+1} mpk_{t+1})
$$

$$
1 + r_t = \frac{1}{1 + \delta} \left( \frac{E_{inv_{t+1}}}{k_{t+1}} \right)^{\Theta_2} + \frac{rmc * mpk}{(1 + \delta) \Theta_2 C_1} E_t rmc_{t+1} + \frac{1}{(1 + \delta) \Theta_2 C_1} (E_t mpk_{t+1} - \delta - r_t) + \frac{\delta}{(1 + \delta)} \hat{k}_t .
$$

Using the log approximation, (3.31) yields the following semi-log-linear expectational investment equation

$$
\hat{inv}_t = \frac{1}{1 + \delta} E_t \hat{inv}_{t+1} + \frac{rmc * mpk}{(1 + \delta) \Theta_2 C_1} E_t rmc_{t+1} + \frac{1}{(1 + \delta) \Theta_2 C_1} (E_t mpk_{t+1} - \delta - r_t) + \frac{\delta}{(1 + \delta)} \hat{k}_t ,
$$

where $\hat{inv} = inv / \overline{inv}$, with $\overline{C_1} = \Theta_1 (\Theta_2 + 1) \delta^{\Theta_2}$. Steady-state adjustment costs are denoted by $\overline{C_1}$; $rmc$, $mpk$ and $inv$ are the steady-state values of real marginal cost, marginal product of capital and investment respectively. The gap between the expected net return on capital and the return on the financial asset is denoted by $E_t mpk_{t+1} - \delta - r_t$. Equation (3.32) shows that current investment depends not only on current, but also on the expected future premiums on investment in real

---

97 See equation (3.12) in Subsection 3.1.
98 For steady-state analysis, see Appendix.
assets because of the inclusion of the forward-looking term $E_t \hat{inv}_{t+1}$. For simplicity, a parameter $\Xi$ denoting the semi-elasticity of investment with respect to the real asset’s premium in (3.32) can be introduced as $\Xi = 1/(1+\delta)\Theta_2 \tilde{C}_1$. Thus, investment behaviour is described by

$$\hat{inv}_t = \frac{1}{1+\delta} E_t \hat{inv}_{t+1} + \Xi \left( rmc\times mpk \times E_t \hat{rmc}_{t+1} + E_t \hat{mpk}_{t+1} - \delta - r_t \right) + \frac{\delta}{1+\delta} \hat{k}_t. \quad (3.32')$$

Thus, under endogenous capital and investment adjustment costs, the model consists of an “IS sector” in the form

$$\hat{c}_t = E_t \hat{c}_{t+1} - \sigma^{-1} (r_t - \bar{r}) + \sigma^{-1} (1 - \rho_v) \nu_t. \quad (3.25)$$

$$\hat{inv}_t = \frac{1}{1+\delta} E_t \hat{inv}_{t+1} + \Xi \left( rmc\times mpk \times E_t \hat{rmc}_{t+1} + E_t \hat{mpk}_{t+1} - \delta - r_t \right) + \frac{\delta}{1+\delta} \hat{k}_t. \quad (3.32')$$

$$\hat{rmc}_t = \hat{\omega}_t - \frac{\alpha_{pf}}{1 - \alpha_{pf}} \left( \hat{k}_t - \hat{y}_t \right) - A, \quad (3.33)$$

$$mpk_t = mpk(\hat{y}_t - \hat{k}_t) \quad (3.34)$$

$$\hat{k}_{t+1} = (1-\delta)\hat{k}_t + \delta \hat{inv}_t. \quad (3.35)$$

$$\hat{y}_t = \hat{\omega}_t \hat{c}_t + \hat{\omega}_g \hat{g}_t + \hat{\omega}_{inv} \hat{inv}_t \quad (3.26)$$

where (3.25) describes consumption decisions made by the households as derived in Subsection 3.2, (3.32’) represents investment behaviour by firms, (3.33) and (3.34) are log-linear approximations to real marginal cost and the marginal product of capital for the Cobb-Douglas production function (3.2), (3.35) is a log-linearisation around the steady state of the investment specification (3.36) and (3.26) is the overall resource constraint with investment from Subsection 3.2. The technology shock $A_t$ and the consumption preference shock $\nu_t$ are modelled as AR(1) processes, with $A_t = \rho_A A_{t-1} + \epsilon_A$ and $\nu_t = \rho_v \nu_{t-1} + \epsilon_{\nu_t}$.

### 3.4. Inflation and real wage equations under sticky prices and wages

In this section, I derive the inflation and real wage relations under sticky prices and wages, which will be used in the subsequent determinacy and impulse response analysis. For illustration, the corresponding equations under flexible prices and wages are presented in Appendix. As a first step, I assume that $\eta_p$ is the fixed probability that households cannot adjust their price as in Calvo (1983), so that the first-order condition on price-setting (3.14) becomes

$$E_t \sum_{i=0}^{\infty} \beta_i \eta_p \left[ \lambda_{\tau_{i+1}} Y_{\tau_{i+1}} (1-\theta) P_{\tau_{i+1}}^{\pi} \left( (P_{\tau_{i+1}}^{\tau})^{1-\theta} + \delta_{\tau_{i+1}} Y_{\tau_{i+1}} \theta P_{\tau_{i+1}}^{\pi(1-\theta)} \right) / (P_{\tau_{i+1}}^{\tau})^{\pi} \right] = 0, \quad (3.14')$$
which can be log-linearised to yield an equation describing sluggish price adjustment

\[ \pi_t - \bar{\pi} = \beta \left( E_t \pi_{t+1} - \bar{\pi} \right) + \frac{(1 - \beta \eta_p)(1 - \eta_p)(1 - \alpha_{pp})}{\eta_p(1 - \alpha_{pp} + \alpha_{pp} \theta)} \left( \eta - \eta \right) \]  \hspace{1cm} (3.36)

Similarly to prices, for nominal wages it may be assumed that they cannot be adjusted with a fixed probability \( \eta_w \), so that the nominal wage first-order condition (3.15) is transformed into

\[ E_t \sum_{i=0}^{\infty} \beta^i \eta_w \left[ \lambda_{w_i} \pi_{t+i} \right] = \frac{1 - \theta_w}{1 - \theta_w \beta} \left( W_t^{-\theta_w} / (W_{t+1}^{-\theta_w}) \right) - \theta_w + \sigma_{w_{t+i}} \theta_w W_t^{-\theta_w + 1} / (W_{t+1}^{-\theta_w}) = 0 . \]  \hspace{1cm} (3.15')

Log-linearising (3.15') yields an expression for the dynamic real wage behaviour under sticky prices of the form

\[ \hat{\omega}_t = \frac{1}{1 + \beta} \hat{\omega}_{t-1} + \frac{\beta}{1 + \beta} E_t \hat{\omega}_{t+1} - \frac{1}{1 + \beta} \left( \pi_t - \bar{\pi} \right) + \frac{\beta}{1 + \beta} \left( E_t \pi_{t+1} - \bar{\pi} \right) + \frac{a}{1 + \beta} \hat{a}_t, \]  \hspace{1cm} (3.37)

with \( a = (1 - \beta \eta_w)(1 - \eta_w) / \eta_w \left[ 1 + \theta_w \tau (\bar{n} / \bar{t}) \right] \) and \( \hat{a}_t \), denoting the log deviations from steady state of the ratio of the average leisure-consumption marginal rate of substitution over the real wage or the left-hand side in (A3.10).\(^{100}\)

### 3.5. Interest-rate rule specifications

This subsection concludes the derivation of the New Keynesian model with endogenous capital and capital adjustment costs by adding a monetary policy rule. For the sake of realism, monetary policy is assumed to be implemented through a nominal interest rate instrument, so that money supply becomes an endogenous variable.\(^{101}\) Policy behaviour can be specified in terms of a Taylor rule as in Taylor (1993):

\[ i_t = r + \pi_t + \lambda_x (\pi_t - \pi) + \lambda_y \hat{y}_t + u_t, \]  \hspace{1cm} (3.38)

\[ \lambda_x, \lambda_y > 0, \]

where \( i_t \) is the nominal interest rate, \( \pi_t \) denotes the inflation rate, \( \hat{y}_t \) is the deviation of actual output from its steady-state (potential) level, \( \bar{r} \) is the steady-

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100 See Appendix. Sbordone (2001) offers a detailed derivation of this real wage equation under sticky wages.

101 The money-demand relation is derived in Appendix. However, with the nominal interest rate chosen as a monetary policy instrument and with a separable period utility function (3.22), the LM equation can be excluded from the model relations that will be calibrated in Subsection 4.1 and used for the subsequent determinacy and impulse response analysis.
state real interest rate and \( \pi^* \) is the central bank’s target inflation rate\(^{102}\). The term \( \varepsilon_i \) stands for is a monetary policy unit shock\(^{103}\). The reaction coefficients \( \lambda_x \) and \( \lambda_y \) determine how strongly the monetary authority stresses inflation and output stabilisation respectively. The response to inflation deviations in (3.38) is actually \( \lambda_x^* = 1 + \lambda_x > 1 \) for \( \lambda_x > 0 \). Thus, in the classification of Leeper (1991), the Taylor rule as in (3.38) is an “active” monetary policy rule, describing a more than one-on-one increase in the policy instrument as a result of deviation of the actual inflation rate from the target (steady-state) rate. Another possible specification of policy behaviour is given by a rule proposed by Casares and McCallum (2006)

\[
i_t - \bar{i} = (1 - \lambda_i) \left[ (1 + \lambda_x)(\pi_t - \bar{\pi}) + \lambda_y \hat{y}_t \right] + \lambda_i \left( i_{t-1} - \bar{i} \right) + \varepsilon_i ,
\]

(3.39)

and

\[
\lambda_x^* = (1 - \lambda_i)(1 + \lambda_x) ; \quad \lambda_y^* = (1 - \lambda_i)\lambda_y ,
\]

(3.40)

whereby \( \lambda_x^* \) and \( \lambda_y^* \) are the effective inflation and output gap response coefficients implied by the policy rule. Additionally, the real interest rate equals the difference between the nominal interest rate and next period’s expected inflation in accordance with the Fisher equation, i.e.

\[
r_t = i_t - E_t \pi_{t+1} .
\]

(3.41)

Equations (3.25), (3.26), (3.32’) and (3.33)-(3.35), together with (3.36), (3.37), (3.39) or (3.38) and (3.41) determine time paths for the ten endogenous variables in the model: \( \hat{c}_t , \hat{inv}_t , \hat{mpk}_t , \hat{rmc}_t , \hat{k}_{t+1} , \hat{y}_t , r_t , \pi_t , \hat{\omega}_t \) and \( i_t \).

4. Determinacy analysis

After the New Keynesian model with endogenous capital and adjustment costs has been derived in the previous section, I proceed with studying the system’s determinacy properties under several policy rule specifications. Since, when deriving the model’s reduced forms, the ten system equations yield quite complex coefficients, it is not possible to use the analytical approach to assessing the system’s determinacy properties. Thus, as a second-best solution, the eigenvalues

\(^{102}\) In the following subsections instead of the target rate \( \pi^* \) the steady-state inflation rate \( \bar{\pi} \) enters the Taylor rule, reflecting the assumption that the central bank correctly assesses the steady-state inflation rate and uses it as a target value.

\(^{103}\) The term \( \varepsilon_i \) is not included in the original Taylor (1993) version, but is added here in order to enable analysis of the impact of a monetary policy shock on the system under different policy specifications. The monetary policy shock term captures nominal interest rate changes that are not a result of the central bank’s response to the target variables as prescribed by the rule. The monetary policy unit shock is modelled as an upward blip of 1 percent in the shock term \( \varepsilon_i \) in the policy rules (3.38) and (3.39) that is transmitted to the policy instrument, causing it to rise.
of the system are examined after substituting the coefficients with the numerical values provided in Subsection 4.1. Then, I offer some preliminary insights to the relevance of the Taylor principle using the calibration in the previous subsection for active and passive rules with and without an output target. Finally, in the last subsection a more global perspective to determinacy outcomes under a wide range of inflation and output gap response coefficients is provided.

4.1. Calibration

In order to enable quantitative analysis of the model’s properties it should be specified in numerical terms. Table 3.1 presents the values of the model parameters\textsuperscript{104}, used for the determinacy analysis in Subsection 4.2 and the impulse responses in Chapter IV. In addition, some of the calibration choices should be considered in detail. For equation (3.25), $\sigma = 5$ implies an intertemporal elasticity of substitution in consumption $\sigma^{-1} = 0.2$ as in Hall (1988) and Fuhrer (2000). The value of the steady-state real interest rate $\bar{r} = 0.005$ corresponds to an annual value of 2 percent. In equation (3.32'), $A_\delta = 5363$ and $A_\gamma = 3.14$ yield $C = \Theta_1 \delta^{\Theta_1} = 0.05$, implying that the unit adjustment cost in steady state equals 5 percent of investment. The marginal adjustment cost in steady state is then $C_\delta = \Theta_1 (\Theta_2 + 1) \delta^{\Theta_2} = 0.21$. The steady-state values of real marginal cost are derived from the relation for the steady-state marginal product of capital (A3.6)\textsuperscript{105}. In equations (3.25) and (3.33), the autocorrelation coefficients assigned to the preference and technology shocks $\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_{\nu_t}$ and $A_t = \rho_A A_{t-1} + \varepsilon_A$ are $\rho_\nu = 0.3$ and $\rho_A = 0.95$.

Table 3.1: Parameter values (in order of appearance)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>5</td>
</tr>
<tr>
<td>$\rho_\nu$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>0.005</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>$\Theta_1$</td>
<td>5363</td>
</tr>
<tr>
<td>$\Theta_2$</td>
<td>3.14</td>
</tr>
<tr>
<td>$\Xi$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\overline{rmc}$</td>
<td>0.83</td>
</tr>
<tr>
<td>$mpk$</td>
<td>0.04</td>
</tr>
<tr>
<td>$\alpha_{pf}$</td>
<td>0.36</td>
</tr>
</tbody>
</table>

\textsuperscript{104} The parameter values chosen are based on the calibration in Casares and McCallum (2006).

\textsuperscript{105} See Appendix.
<table>
<thead>
<tr>
<th>Equation</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate supply (3.36):</td>
<td>$\bar{\pi} = 0.005$, $\beta = 0.99$, $\theta = 6$, $\eta_p = 0.75$</td>
</tr>
<tr>
<td>Real wage equation (3.37):</td>
<td>$\theta_W = 4$, $\eta_W = 0.75$, $\tau = 2$, $\bar{\eta}/\bar{l} = 0.5$</td>
</tr>
<tr>
<td>Monetary policy rule (3.38), active</td>
<td>$\lambda_\pi = 0.5$, $\lambda_\pi^* = 1.5$, $\lambda_\gamma = \lambda_\gamma^* = 0.5$</td>
</tr>
<tr>
<td>Monetary policy rule (3.38), passive</td>
<td>$\lambda_\pi^* = 0.5$, $\lambda_\gamma = \lambda_\gamma^* = 0.5$; $\lambda_\gamma = \lambda_\gamma^* = 0.1$&lt;sup&gt;106&lt;/sup&gt;</td>
</tr>
<tr>
<td>Monetary policy rule (3.39):</td>
<td>$\lambda_\pi = 1.5$, $\lambda_\pi^* = 0.3$, $\lambda_\gamma = 0.1$, $\lambda_\gamma^* = 0.02$, $\lambda_\gamma^* = 0.8$</td>
</tr>
</tbody>
</table>

In the aggregate supply equation (3.36) $\bar{\pi} = 0.005$ corresponds to an annual value of steady-state inflation of 2 percent. The degree of price and wage stickiness is represented by the probability that the representative household is not able to adjust its price or wage; $\eta_p = \eta_w = 0.75$ implies that prices and wages are reset once a year on average<sup>107</sup>. The value of the elasticity of substitution between differentiated consumption goods $\theta = 6$ is frequently assigned in the business-cycle literature. Thus some simple calculations show that in (3.36) the coeffi-

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106 In Chapter III, Subsection 4.2 the value $\lambda_\gamma = \lambda_\gamma^* = 0.1$ has been used for the determinacy analysis. Based on the finding that such a value of the output gap coefficient cannot yield a determinate rational-expectations equilibrium, the value $\lambda_\gamma = \lambda_\gamma^* = 0.5$ has been chosen for the impulse-response analysis in Chapter IV, Subsections 3.2 and 3.3.

107 As in Erceg et al. (1999).
cient \((1 - \beta \eta_p)(1 - \eta_p)(1 - \alpha_{pp}) / \eta_p(1 - \alpha_{pp} + \alpha_{pp} \theta)\) is equal to 0.02, i.e. the deviations of real marginal cost have a relatively modest impact on inflation dynamics, especially in comparison to the influence of inflation expectations. In the wage adjustment equation (3.37) the leisure relative risk aversion coefficient \(\tau = 2\) implies that in the steady state leisure is twice the labour hours \((\tilde{n} / \tilde{l} = 0.5)\). When calibrating the monetary policy rule specifications (3.38) and (3.39), I have included for convenience the terms \(\lambda^*_i\) and \(\lambda^*_y\), which denote the effective inflation and output gap response coefficients implied by the policy rule.

In the forthcoming determinacy and shock response analysis two main simple monetary rule specifications will be tested, namely an “active” and a “passive” monetary policy rule. The former possibility can be illustrated by a baseline Taylor rule that incorporates an active policy stance with regard to the inflation objective and a more moderate response to output fluctuations as in (3.38). The latter option can be represented by a passive Taylor rule with a reaction coefficient assigned to inflation smaller than unity (for instance, 0.5) and no interest-rate smoothing as in (3.38). The passive policy stance can also be illustrated by a rule of the form (3.39), proposed by Casares and McCallum (2006), where the effective coefficients assigned to inflation and output deviations are considerably smaller than in the previous case (0.3 and 0.02 respectively) and interest-rate smoothing has a considerable weight for setting the policy instrument.

The above mentioned options for the design of a simple monetary policy rule can be reduced to three main issues to be explored. Firstly, whether the Taylor principle requiring an active policy stance with respect to inflation still needs to be adhered to in a framework with endogenous capital, capital adjustment costs and sticky prices and wages. Secondly, whether the introduction of output-targeting ensures determinacy of rational expectations equilibrium (REE) and/or can alleviate the negative effects of shocks. Thirdly, whether interest-rate smoothing can compensate for the impact of non-compliance with the Taylor principle in the case of passive policy stance or, alternatively, improve the results attained by active policy. In order to enable comparability of the results concerning the effect of the degree of “activism”, response to output fluctuations and interest-rate smoothing, six cases will be considered: active rule with inflation-targeting only, active rule with inflation- and output-targeting and no interest-rate smoothing (as in the Taylor rule), active rule with inflation- and output-targeting and interest-rate smoothing, passive rule with inflation-targeting only, passive rule with inflation- and output-targeting and no interest-rate smoothing (as in the Taylor rule) and finally a passive rule with inflation- and output-targeting and interest-rate smoothing. In the case of passive policy, experi-

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108 In order to limit the dimensions of the reduced-form system, in the determinacy analysis in Chapter III, Subsection 4.2 interest-rate smoothing is abstracted from. Later on, for the impulse-response analysis in Chapter IV the interest-rate smoothing option is included.
menting with the size of the inflation and output response coefficients provides additional insights to whether the magnitude of the monetary authority’s response when passive policy is pursued has a significant impact on the results obtained. For this purpose, in Chapter IV, Subsection 3.3 I include impulse response results under both (3.38) entailing a relatively stronger passive response with inflation and output coefficients of 0.5 each and (3.39) implying a weaker reaction, with parameter values of 0.3 and 0.02 respectively.

4.2. Determinacy and the Taylor principle: some numerical examples

The structural equations (3.25), (3.26), (3.32’) and (3.33)-(3.35), together with (3.36), (3.41), (3.39) or (3.38) and (3.41) can be reduced to a system of four equations, expressed in terms of inflation, consumption, capital and the real wage. Unfortunately, when deriving the four reduced forms, the ten system equations yield quite complex coefficients, which makes the analytical approach to assessing the system’s determinacy properties impossible. Thus, as a second-best solution, the eigenvalues of the system will be studied after substituting the coefficients with the numerical values included in the previous subsection. First, in the next two subsections I offer some preliminary insights using the calibration in the previous subsection for active and passive rules with and without an output target. Then, in Subsection 4.2.3 a more global perspective to determinacy outcomes under a wide range of inflation and output gap response coefficients is provided.

For convenience, the system’s reduced forms are presented as:

\[\hat{k}_{t+1} = a_1 \hat{k}_{t+1} + a_2 \hat{k}_t - a_3 \pi_t - a_4 \hat{c}_t - a_5 \hat{\omega}_t + a_6 \hat{\omega}_{t-1}\]  
(A3.13)

\[E_t \pi_{t+1} = -a_7 \hat{k}_{t+1} + a_8 \hat{k}_t + a_9 \pi_t - a_{10} \hat{c}_t - a_{11} \hat{\omega}_t\]  
(A3.14)

\[E_t \hat{\omega}_{t+1} = a_{12} \hat{k}_{t+1} - a_{13} \hat{k}_t + a_{14} \hat{c}_t + a_{15} \hat{\omega}_t - a_{16} \hat{\omega}_{t-1}\]  
(A3.15)

\[E_t \hat{c}_{t+1} = a_{17} \hat{k}_{t+1} - a_{18} \hat{k}_t + a_{19} \pi_t + a_{20} \hat{c}_t + a_{21} \hat{\omega}_t\]  
(A3.16)

The system (A3.13)-(A3.16) can be written in the vector form \(E_t X_{t+1} = AX_t\). If the identities \(\hat{k}_{t+1} = \hat{k}_{t+1}\) and \(\hat{\omega}_t = \hat{\omega}_t\) are added to the system, vector \(X_t\) contains the six elements \(\hat{k}_{t+1}, \hat{k}_t, \pi_t, \hat{c}_t, \hat{\omega}_t\) and \(\hat{\omega}_{t-1}\) of which only the capital stock \(k\) is a pre-determined state variable.

109 The step-by-step derivation, as well as the analytical form of the reduced forms’ coefficients, are included in Appendix.
Under the parameter values chosen in Chapter III, Subsection 4.1\textsuperscript{110},

\[ a_1 = 1.99 + 0.33 \lambda_y^* , \]
\[ a_2 = -(0.98 + 0.32 \lambda_y^* ) , \]
\[ a_3 = 0.04 - 0.04 \lambda_z^* , \]
\[ a_4 = 0.001 - 0.03 \lambda_y^* , \]
\[ a_5 = 0.002 , \]
\[ a_6 = 0.001 , \]
\[ a_7 = 0.1 , \]
\[ a_8 = 0.11 , \]
\[ a_9 = 1.01 , \]
\[ a_{10} = 0.01 , \]
\[ a_{11} = 0.02 , \]
\[ a_{12} = 0.1 , \]
\[ a_{13} = 0.11 , \]
\[ a_{14} = 0.01 , \]
\[ a_{15} = 2.03 , \]
\[ a_{16} = a_9 = 1.01 , \]
\[ a_{17} = 0.02 + 1.76 \lambda_y^* , \]
\[ a_{18} = 0.02 + 1.72 \lambda_y^* , \]
\[ a_{19} = 0.2 \lambda_z^* - 0.2 , \]
\[ a_{20} = 1.002 + 0.16 \lambda_y^* , \]
\[ a_{21} = 0.004 . \]

\textsuperscript{110} Here the parameters \( a_5 = 0.002 , a_6 = 0.001 \) and \( a_{21} = 0.004 \) will be taken as equal to zero, as all three of them are significantly smaller than 0.01.
4.2.1. Active rule

The case with inflation-targeting only

For an active interest rate rule with inflation-targeting only with \( \lambda^*_z = 1.5 \) and \( \lambda^*_y = 0 \), the coefficient matrix \( F \) is given by:

\[
F = \begin{bmatrix}
1.99 & -0.98 & 0.02 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
-0.1 & 0.11 & 1.01 & -0.01 & -0.02 & 0 \\
0.02 & -0.02 & 0.1 & 1 & 0 & 0 \\
0.1 & -0.11 & 0 & 0.01 & 2.03 & -1.01 \\
0 & 0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}.
\]

Again, only \( \hat{k}_t \) is a pre-determined state variable, which implies that for determinacy of rational expectations equilibrium \( F \) should have exactly one eigenvalue inside the unit circle and the remaining five eigenvalues should be greater than unity. In order to guarantee methodical consistence, coefficient values are represented with a 0.01 accuracy as in the previous subsection. Thus \( a_5 = 0.002 \), \( a_6 = 0.001 \) and \( a_{21} = 0.004 \) are set to equal zero and \( a_{20} = 1.002 \) is taken to be equal to 1. Since after this procedure the dynamics of the capital stock, inflation and consumption are independent of the realisations of the real wage in \( t-1 \) and one eigenvalue of \( F \) is given by \( a_{16} > 1 \), stability of the system is completely determined by the 5x5 submatrix \( \hat{F} \):

\[
\hat{F} = \begin{bmatrix}
1.99 & -0.98 & 0.02 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
-0.1 & 0.11 & 1.01 & -0.01 & -0.02 \\
0.02 & -0.02 & 0.1 & 1 & 0 \\
0.1 & -0.11 & 0 & 0.01 & 2.03 \\
\end{bmatrix}.
\]

The eigenvalues of \( \hat{F} \) are given by \((0.91, 0.997+0.03i, 0.997-0.03i, 1.1, 2.03)\), of which the first three eigenvalues of the system are within the unit circle. Thus, an active rule satisfying the Taylor principle under inflation-targeting only does not necessarily yield determinacy of rational expectations equilibrium in a model with endogenous capital and investment under the parameter values chosen.

The case with inflation- and output-targeting

For an active interest rate rule with \( \lambda^*_z = 1.5 \) and \( \lambda^*_y = 0.5 \), the coefficient matrix \( G \) is given by:

---

111 The same parameter values are assigned in Taylor (1993).
As in the system of equations (3.42) only $\hat{k}_t$ is a pre-determined state variable, the necessary and sufficient condition for determinacy of rational expectations equilibrium is that $G$ should have exactly one eigenvalue inside the unit circle and the remaining five eigenvalues greater than unity. By analogy to the previous case, since the dynamics of the capital stock, inflation and consumption are independent of the realisations of the real wage in $t-1$ and one eigenvalue of $G$ is given by $a_{ik}>1$, stability of the system is completely determined by the 5x5 submatrix $\hat{G}$:

$$
\hat{G} = \begin{bmatrix}
2.16 & -1.14 & 0.02 & -0.01 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
-0.1 & 0.11 & 1.01 & -0.01 & -0.02 & 0 \\
0.9 & -0.88 & 0.1 & 1.08 & 0 & 0 \\
0.1 & -0.11 & 0 & 0.01 & 2.03 & -1.01 \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
$$

The eigenvalues of $\hat{G}$ are given by $(0.94, 1.01, 1.52 - 0.02i, 1.52 + 0.02i, 2.03)$, of which only the first eigenvalue of the system is within the unit circle. Thus, an active rule satisfying the Taylor principle can yield a determinate rational expectations equilibrium when both inflation and output enter the interest rate rule.

### 4.2.2. Passive rule

*The case with inflation-targeting only*

For a passive interest rate rule with inflation-targeting only with $\lambda'_z = 0.5$ and $\lambda'_y = 0$, the coefficient matrix $H$ is given by:

$$
H = \begin{bmatrix}
1.99 & -0.98 & -0.02 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
-0.1 & 0.11 & 1.01 & -0.01 & -0.02 & 0 \\
0.02 & -0.02 & -0.1 & 1 & 0 & 0 \\
0.1 & -0.11 & 0 & 0.01 & 2.03 & -1.01 \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
$$

Again, only $\hat{k}_t$ is a pre-determined state variable, which implies that for determinacy of rational expectations equilibrium $H$ should have exactly one eigenvalue inside the unit circle and the remaining five eigenvalues greater than unity. In order to guarantee methodical consistence coefficient values are represented...
with an 0.01 accuracy as in the previous subsection. Thus \( a_5 = 0.002, a_6 = 0.001 \) and \( a_{21} = 0.004 \) are set to equal zero and \( a_{20} = 1.002 \) is taken to be equal to 1. Since after this procedure the dynamics of the capital stock, inflation and consumption are independent of the realisations of the real wage in \( t-1 \) and one eigenvalue of \( H \) is given by \( a_{10} > 1 \), stability of the system is completely determined by the 5x5 submatrix \( \tilde{H} \):

\[
\tilde{C} = \begin{bmatrix}
1.99 & -0.98 & -0.02 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
-0.1 & 0.11 & 1.01 & -0.01 & -0.02 \\
0.02 & -0.02 & -0.1 & 1 & 0 \\
0.1 & -0.11 & 0 & 0.01 & 2.03 \\
\end{bmatrix}.
\]

The eigenvalues of \( \tilde{H} \) are given by \((0.88, 0.98, 1.05, 1.1, 2.03)\), of which the first and the second eigenvalues of the system are within the unit circle. Thus, a passive rule under inflation-targeting only yields indeterminacy of rational expectations equilibrium under the parameter values chosen.

*The case with inflation- and output-targeting*

For a passive interest rate rule with \( \lambda_x^* = 0.5 \) and \( \lambda_y^* = 0.1 \), the coefficient matrix \( J \) is given by:

\[
J = \begin{bmatrix}
2.02 & -1.01 & -0.02 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
-0.1 & 0.11 & 1.01 & -0.01 & -0.02 \\
0.2 & -0.2 & -0.1 & 1.02 & 0 \\
0.1 & -0.11 & 0 & 0.01 & 2.03 & -1.01 \\
0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}.
\]

Again, as in the system of equations (3.42) only \( \hat{K}_r \) is a pre-determined state variable, the necessary and sufficient condition for determinacy of rational expectations equilibrium is that \( J \) should have exactly one eigenvalue inside the unit circle and the remaining five eigenvalues greater than unity. Since the dynamics of the capital stock, inflation and consumption are independent of the realisations of the real wage in \( t-1 \) and one eigenvalue of \( J \) is given by \( a_{10} > 1 \), stability of the system is completely determined by the 5x5 submatrix \( \hat{J} \):

\[
\hat{J} = \begin{bmatrix}
2.02 & -1.01 & -0.02 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
-0.1 & 0.11 & 1.01 & -0.01 & -0.02 \\
0.2 & -0.2 & -0.1 & 1.02 & 0 \\
0.1 & -0.11 & 0 & 0.01 & 2.03 \\
\end{bmatrix}.
\]

\[112\] See the calibration of a passive rule under (3.38).
The eigenvalues of $\hat{J}$ are given by (0.89, 0.99, 1.05, 1.11, 2.03), of which the first and the second eigenvalues of the system are within the unit circle\textsuperscript{113}. Thus, a passive rule with $\lambda_x^* = 0.5$ and $\lambda_y^* = 0.1$, cannot yield a determinate rational expectations equilibrium when both inflation and output enter the interest rate rule.

4.2.3. Interest-rate rule response coefficient values and determinacy: a global perspective

The previous numeric examples reveal differing stability properties of the system when certain numeric values of the monetary policy response coefficients are assumed. Subsection 4.2.1 shows that adherence to the Taylor principle alone does not guarantee determinacy of rational expectations equilibrium under endogenous capital, as the output gap coefficient also plays an important role. Therefore, it is necessary to examine the results for a greater number of parameter values within a plausible interval, in order to identify the stability regions for the values of $\lambda_x^*$ and $\lambda_y^*$.

More generally, the system’s coefficient matrix can be represented in terms of both policy parameters from the interest rate rule as:

$$
\begin{bmatrix}
1.99 + 0.33\lambda_x^* & -0.98 - 0.32\lambda_y^* & 0.04\lambda_x^* - 0.04 & 0.03\lambda_y^* - 0.001 & 0 \\
1 & 0 & 0 & 0 & 0 \\
-0.1 & 0.11 & 1.01 & -0.01 & -0.02 \\
1.76\lambda_x^* + 0.02 & -1.72\lambda_y^* - 0.02 & 0.2\lambda_x^* - 0.2 & 0.16\lambda_y^* + 1.002 & 0 \\
0.1 & -0.11 & 0 & 0.01 & 2.03 \\
\end{bmatrix}.
$$

Fig. 3.1 shows in three-dimensional space the determinacy results for values of the policy response parameters $\lambda_x^*$ and $\lambda_y^*$ from 0 to 2\textsuperscript{114}. The vertical axis represents the unit circle. Whenever only one eigenvalue of the system is smaller than unity, the respective combination of values of $\lambda_x^*$ and $\lambda_y^*$ is marked in blue. Alternatively, the cases when the parameter values generate less or more than one eigenvalue within the unit circle are denoted by blank space. The intersection of the $\lambda_x^*$ and $\lambda_y^*$ axes with the unit circle (1.0) plane is represented from a two-dimensional perspective on Fig. 3.2, where the black regions stand for indeterminacy of rational expectations equilibrium (none or two or more eigenvalues within the unit circle) and the white areas correspond to parameter values inducing a unique convergence path with a single eigenvalue smaller than unity.

Figures 3.1 and 3.2 reveal a picture that is consistent with the numeric results in the previous section. With endogenous capital, the introduction of output-targeting is crucial for the determinacy of rational expectations equilibrium.

\textsuperscript{113} The exact value of the second eigenvalue of the system is 0.99341, i.e. very close to unity, but still within the unit circle.

\textsuperscript{114} For the purpose of the analysis, the values of $\lambda_x^*$ and $\lambda_y^*$ are being altered with a 0.05 step.
What is more, adherence to the Taylor principle alone does not necessarily guarantee a unique convergence path of the system when capital is endogenous. Only within a very limited value interval of the inflation reaction coefficient between 1.0 and 1.3 does an active rule with no output gap response yield determinacy. For all other values of $\lambda^*_x$ above 1.3 and under 1.0, setting $\lambda^*_y = 0$ leads to more than one eigenvalue within the unit circle, i.e. to multiple rational expectations equilibria. Still, bearing in mind the above mentioned findings on output-targeting, an active policy in terms of inflation (i.e. $\lambda^*_x > 1$) is more likely to lead to a unique equilibrium, as the required intensity of the output response in this case is significantly smaller than under a passive rule. For example, for $\lambda^*_x = 2$, any $\lambda^*_y$ higher than 0.1 yields determinacy; for values of the inflation coefficient significantly smaller than unity, a much stronger output gap response is needed, e.g. an inflation parameter $\lambda^*_x = 0.5$ requires an output response of an equal size ($\lambda^*_y = 0.5$).

Figure 3.1: Determinacy regions (3D)

Figure 3.2: Determinacy regions (2D)
Apart from the requirement that some small degree of output-targeting should be introduced when $\lambda^*_z$ is above 1.3 in order to guarantee determinacy of rational expectations equilibrium, the above findings in general reaffirm adherence to the Taylor principle as a policy recommendation. Except for a small indeterminacy region, active policy in fact generates determinacy and even unconditionally so for the interval $1 < \lambda^*_z < 1.3$. The second, more controversial, conclusion refers to the implications of passive policy. Contrary to the results of Taylor (1993), in a model specification with endogenous capital $\lambda^*_z < 1$ does not necessarily induce indeterminacy of rational expectations equilibrium. With a sufficient (ever-increasing as $\lambda^*_z$ decreases) degree of an output gap response, a determinate equilibrium may still be obtained even if $\lambda^*_z < 1$.

Generally, the results plotted on Figures 3.1 and 3.2 show that an active interest rate rule might be more advantageous as it is associated with a wider range of response coefficients’ values inducing a unique equilibrium. Determinacy of rational expectations equilibrium is a highly desirable quality, as it assures that, after some disturbance has occurred, the system will converge to steady state following a unique, predictable path. Still, the exact unique adjustment path of the economy and especially the size and the persistence of deviations from steady state also play a crucial role in monetary policy decision-making. For example, a monetary policy stance that guarantees a faster convergence to steady state with smaller and less fluctuating deviations in variables such as interest rates, inflation, the output gap, consumption and investment (to name a few) will clearly be preferred to scenarios generating more uncertainty and distress in the economy. The adjustment paths induced by several different interest rate rule specifications will be presented and discussed in the next chapter.

5. Preliminary summary of results

In Sections 1 and 2, I gave an overview to the baseline New Keynesian framework and possible approaches to modelling capital and investment. Then, in Section 3 I derived a model with endogenous capital, sticky prices and wages and capital adjustment costs. As a next step, different specifications of the interest rate rule with respect to the degree of activeness of the rule (measured by the inflation coefficient) and the target variables included were assessed based on the existence of a determinate rational expectations equilibrium.

The numerical experiments pertaining to the specification of the central bank’s interest rate rule were illustrated by Figures 3.1 and 3.2 that plot determinacy and indeterminacy regions for each combination of values for the inflation and the output gap reaction coefficients $\lambda^*_z$ and $\lambda^*_y$ in the interval $[0,2]$. The determinacy analysis in Section 4 led to the conclusion that, under the assumption of endogenous capital and investment with capital adjustment costs and
calibration of the model parameters consistent with the existing literature\textsuperscript{115}, adherence to the Taylor principle alone does not guarantee determinacy of rational expectations equilibrium under endogenous capital, as the output gap coefficient also plays an important role. Even under an active rule, some degree of output-targeting is needed to guarantee determinacy (except for the very limited value interval of the inflation reaction coefficient between 1.0 and 1.3). Secondly and more interestingly, a passive interest rate rule does not necessarily yield indeterminacy of rational expectations equilibrium. With a significantly strong response of monetary policy to the output gap (ever-increasing as $\lambda^*_z$ decreases), a determinate equilibrium may still be obtained even if the inflation response coefficient is smaller than unity.

The general conclusion from the numerical experiments in Subsection 4.2.3 that an output target improves the performance of policy rules irrespective of the degree of the inflation response is a useful starting point for assessing the results from the shock impulse responses in the next chapter.

\textsuperscript{115} See, for example, Casares and McCallum (2006).